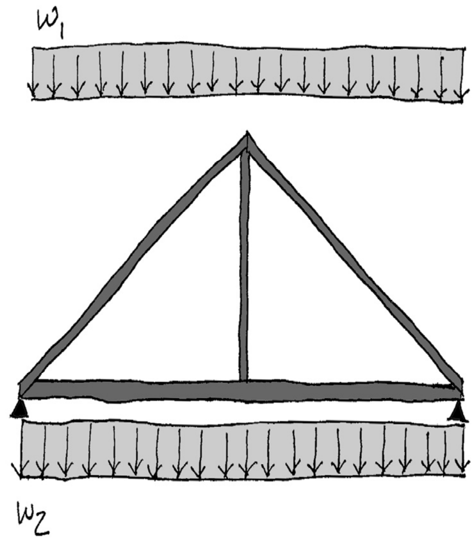
Architecture 314
Structures I

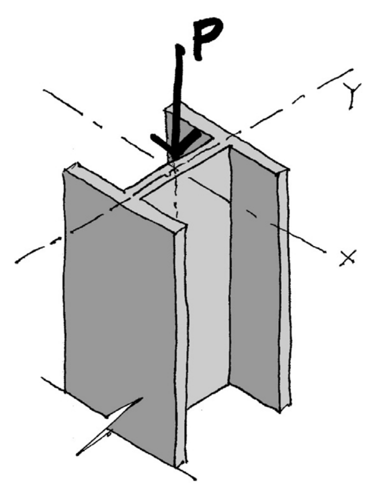
Combined Stress



- Tension + Flexure
- Compression + Flexure
- Eccentric Loads

Axial Stress

- Loads pass through the centroid of the section , i.e. axially loaded
- Member is straight
- Load less than buckling load

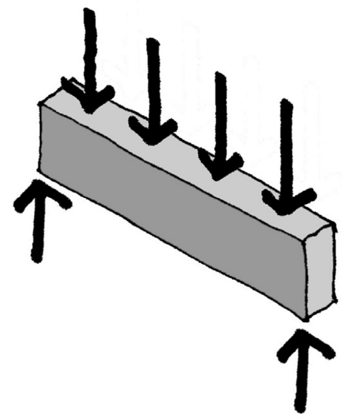


Then:

$$f_a = \frac{P}{A}$$

Flexural Stress

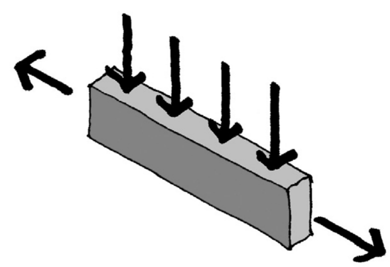
- Loads pass through the **centroid** of the section
- Member is **straight**
- Member deflects in the plane of loading (vertical) – no lateral tensional buckling (LTB)



$$f_b = \frac{Mc}{I}$$

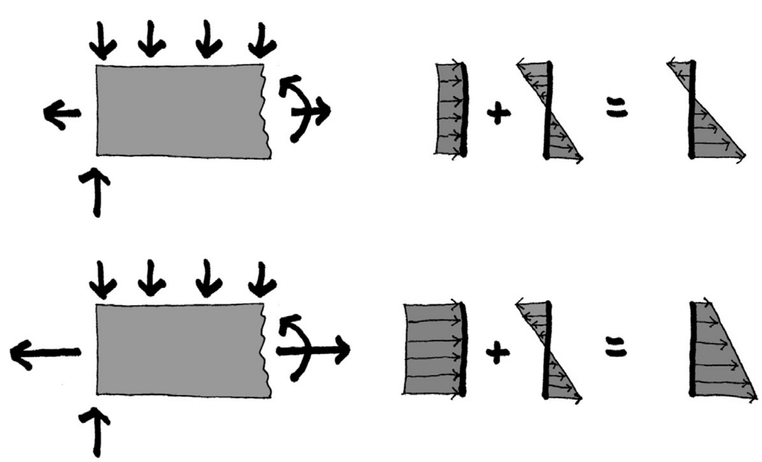
Axial + Flexure

Axial Tension + Flexure



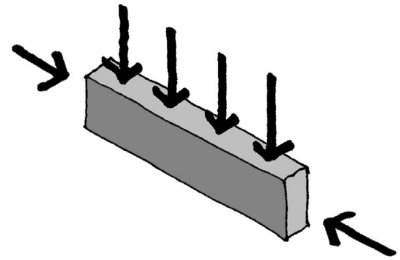
Stress addition by sign:
tension + tension = total
tension – compression = total

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$



Axial + Flexure

Axial Compression + Flexure

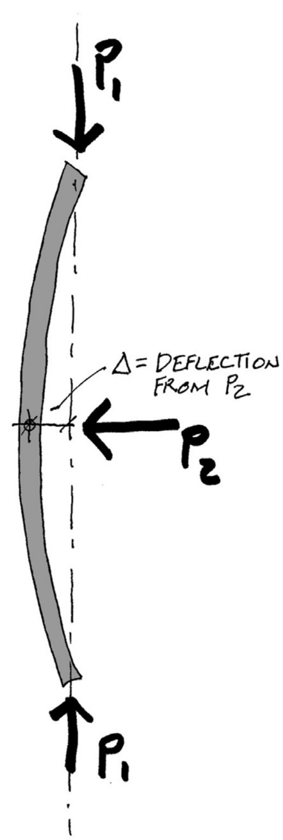


The deflection caused by flexure together with the axial compression results in a secondary moment

$M_2 = P\ \Delta$

$$f = \frac{P}{A} \pm \frac{Mc_x}{I_x} \pm \frac{P\Delta c_x}{I_x}$$

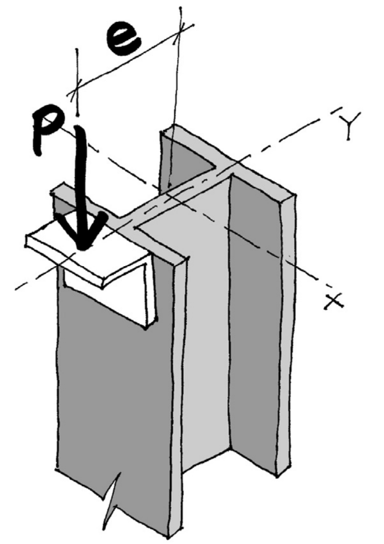
Second Order Stress
"P Delta Effect"



1. Flexure load causes deflection
2. Deflection makes axial load eccentric
3. Eccentric load results in Pe moment
4. Moment causes additional bending
5. More bending increases deflection
6. More deflection increases eccentricity
7. Cycle continues until it either stabilizes or buckles.

Eccentric Loads

- Load offset from centroid
- $M_e = P\ e$
- Total load = P + M_e



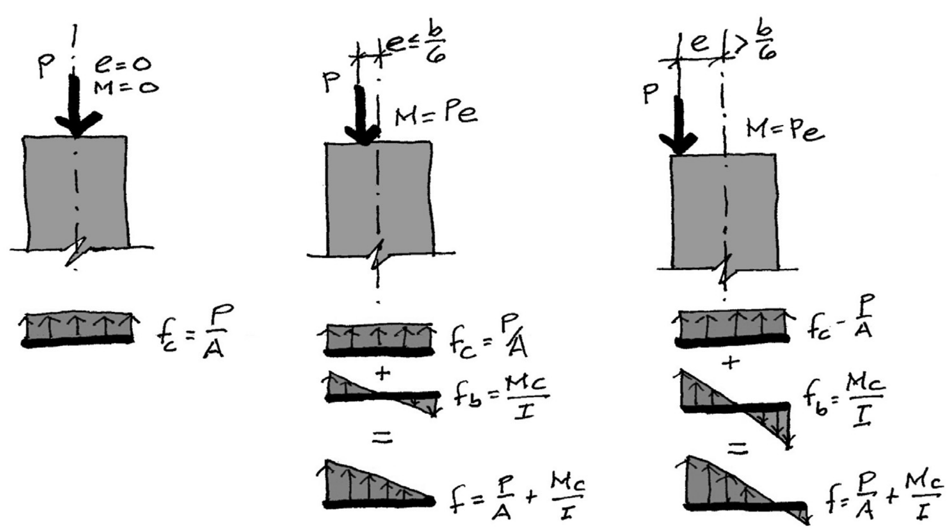
combined stress (interaction) formula:

$$f = \frac{P}{A} \pm \frac{M_e\ c}{I}$$

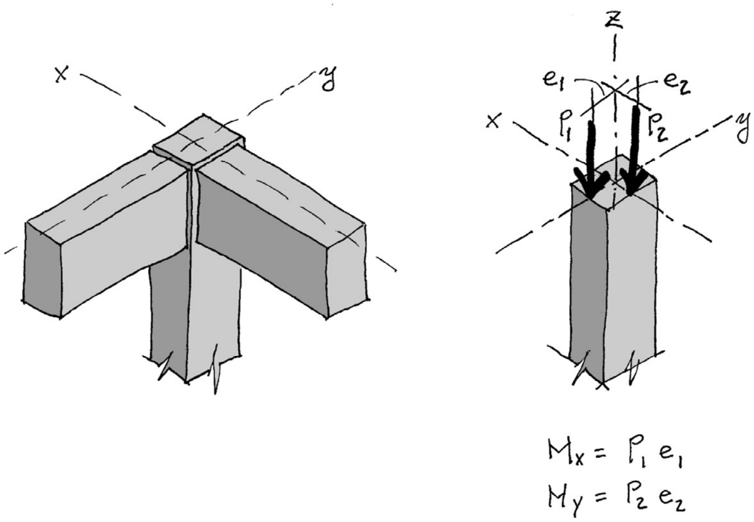
$$\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0$$

Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign



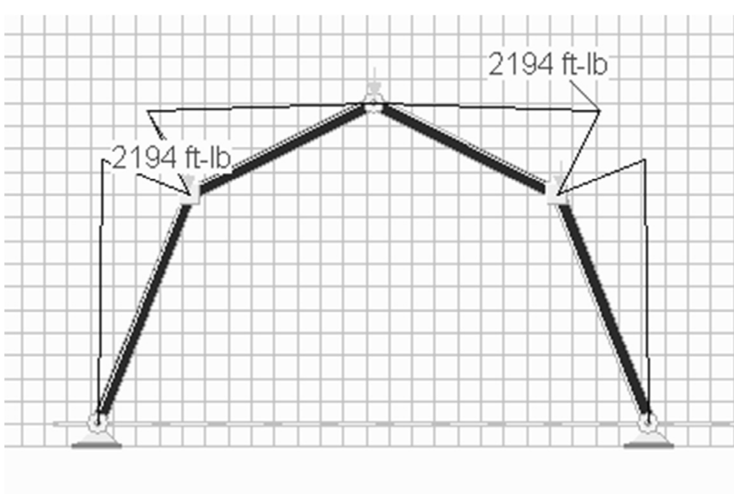
Bi-axial Flexure



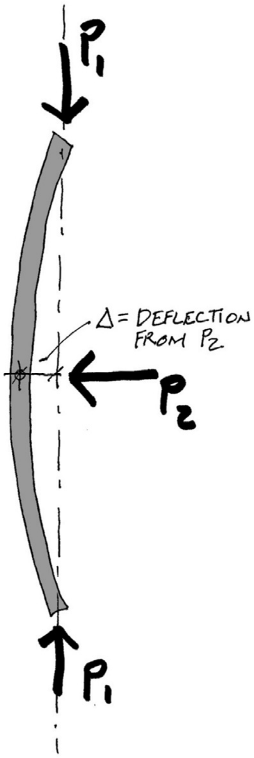
$$f = \frac{P_1}{A} \pm \frac{M_x}{S_x} + \frac{P_2}{A} \pm \frac{M_y}{S_y}$$

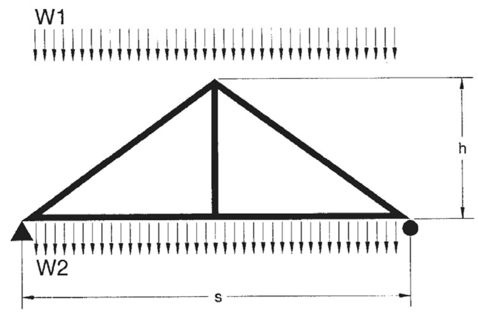
taking tension as negative

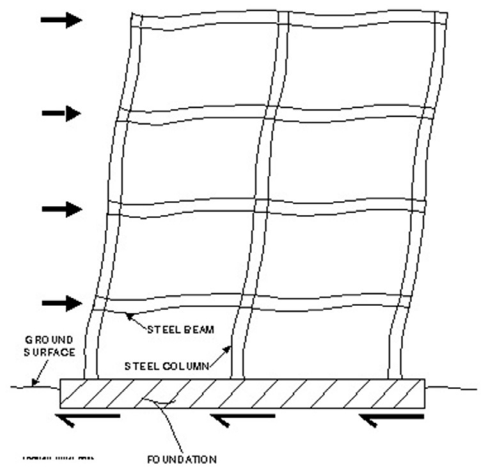
Example of combined Stress Beam Columns



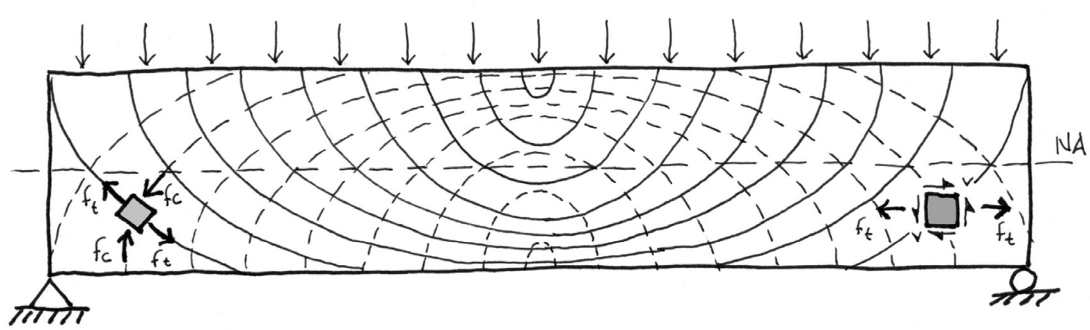
Other Examples

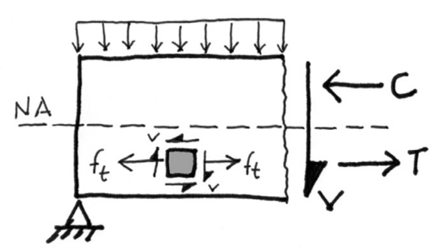


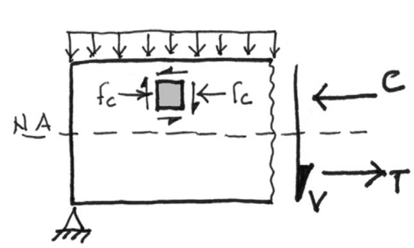




Combined Shear and Bending Stress
Principal Stresses



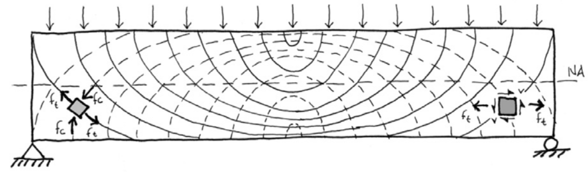


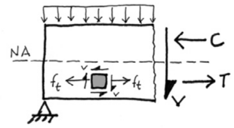


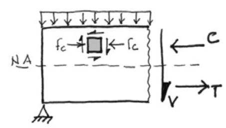
dashed lines follow maximum compression; solid lines maximum tension

Principal Stresses

The surfaces of maximum tension and maximum compression stresses are at right angles, 90°.



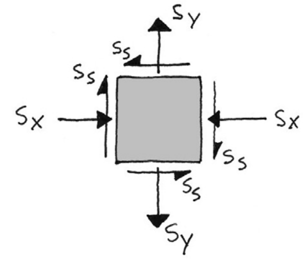




Given the normal and shear stresses on the faces of any elemental square, the principal normal stresses can be calculated by:

$$s'_{N_{\max}} = \frac{s_x + s_y}{2} + \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2}$$

$$s'_{N_{\min}} = \frac{s_x + s_y}{2} - \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2}$$



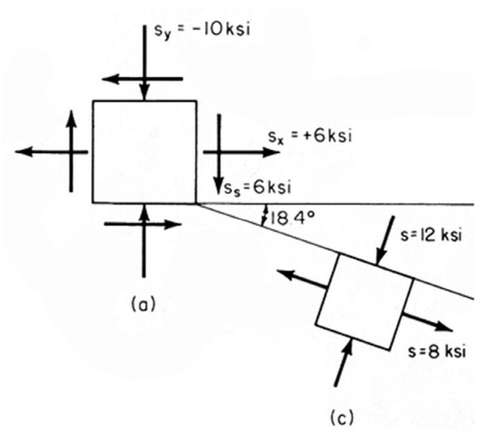
$$\tan 2\theta = -\frac{2s_s}{s_x - s_y}$$

Example (by equations)

$$s'_{N_{\max}} = \frac{s_x + s_y}{2} + \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2}$$

$$s'_{N_{MAX}} = \frac{6-10}{2} + \sqrt{\left(\frac{6+10}{2}\right)^2 + 6^2}$$

$$-2 + \sqrt{8^2 + 6^2} = 8$$



$$\tan 2\theta = -\frac{2s_s}{s_x - s_y}$$

$$s'_{N_{\min}} = \frac{s_x + s_y}{2} - \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_s^2}$$

$$s'_{N_{MIN}} = \frac{6-10}{2} - \sqrt{\left(\frac{6+10}{2}\right)^2 + 6^2}$$

$$-2 - \sqrt{8^2 + 6^2} = -12$$

$$\tan 2\theta = -\frac{2(6)}{6 - -10} = -\frac{12}{16}$$

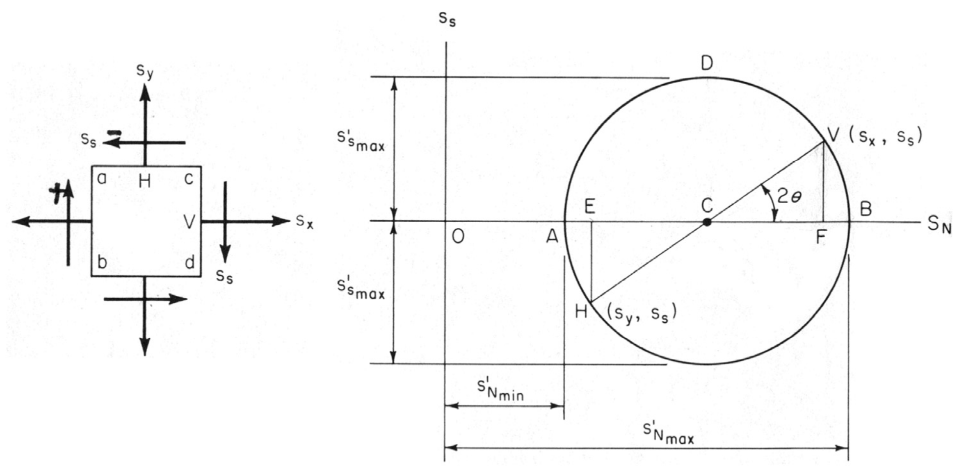
$$= -0.75$$

$$2\theta = -36.87°$$

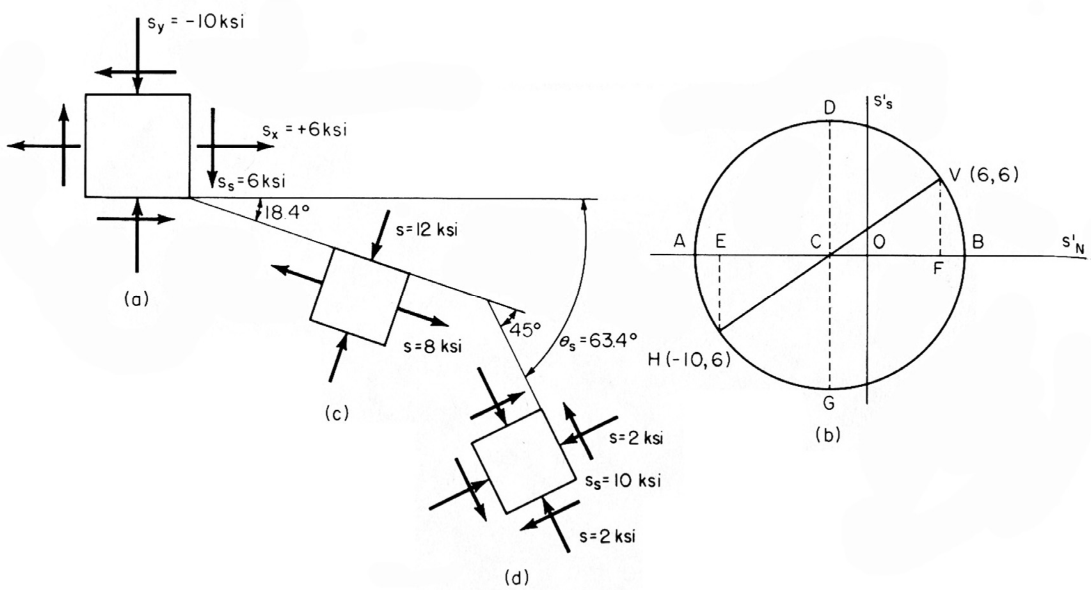
$$\theta = -18.43°$$

Mohr's Circle – Graphic Method to find Principal Stress

1. Choose two adjacent sides of the elemental square (H & V)
2. Plot the coordinates (s_y, s_s) and (s_x, s_s) with S_N as abscissa and S_s as ordinate. Take normal tension stress and clockwise shear stress as positive.
3. Connect the two points with a line and find the center, C
4. Draw a circle with center at C, passing through H and V
5. Calculate tan 2θ = FV/CF
6. Read principal stress values at A and B and max shear stress at D



Mohr's Circle – Example

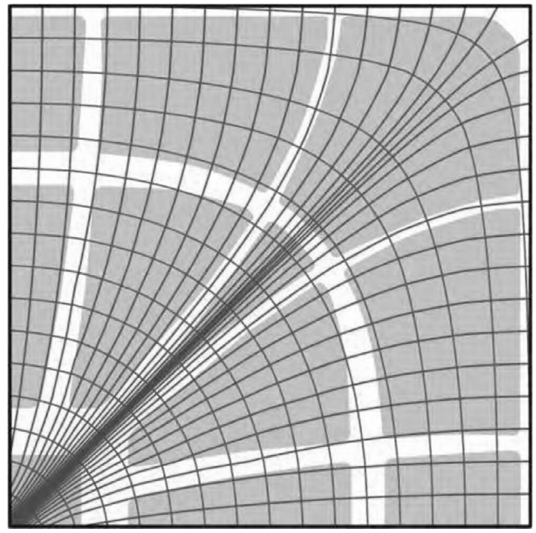


$$OB = -OC + \text{radius of circle} = -OC + \sqrt{(CF)^2 + (FV)^2}$$
$$= -2 + \sqrt{(8)^2 + (6)^2} = -2 + 10$$
$$= +8;$$
$$OA = -OC - \text{radius of circle} = -OC - \sqrt{(CE)^2 + (Eh)^2}$$
$$= -2 - 10$$
$$= -12 \text{ ksi};$$
$$\tan 2\theta = \frac{FV}{CF} = \frac{6}{8} = 0.75, \qquad 2\theta = 36.8°, \qquad \theta = 18.4°$$

$$s_{s_{max}} = 10 \text{ ksi}$$
$$2\theta_s = 2\theta + 90° = 36.8° + 90° = 126.8°$$
$$\theta_s = 63.4°$$

Principal Stresses

Pier Luigi Nervi, Gatti Wool Factory, Rome



Lines of principle stress

Principal Stresses

Pier Luigi Nervi, Palace of Labor Floor System
Palace of Labor (Palazzo del Lavoro)

The Ribbed Floor Slab Systems of Pier Luigi Nervi; Allison B. Halpern, David P. Billington, Sigrid Adriaenssens in "Beyond the Limits of Man" IASS Symposium 2013