

# Plane Trusses Method of Joints

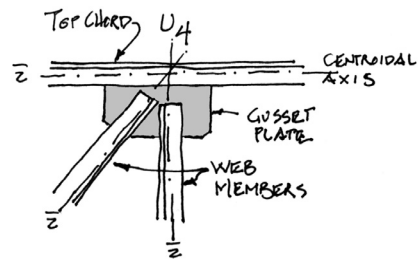
- Definition and Assumptions
- Nomenclature
- Stability and Determinacy
- Analysis by joints



Phaeodaria – Ernst Haeckel

## Definitions and Assumptions

- 2 Force Members
- Pinned Joints
- Concurrent Member Centroids at Joints
- Joint Loaded
- Straight Members
- Small Deflections



Bullring Covering, Xàtiva, Spain  
Kawaguchi and Engineers, 2007

# Nomenclature

## Panels

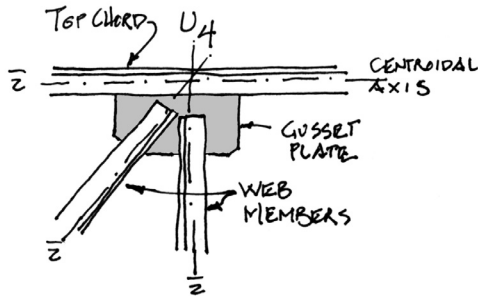
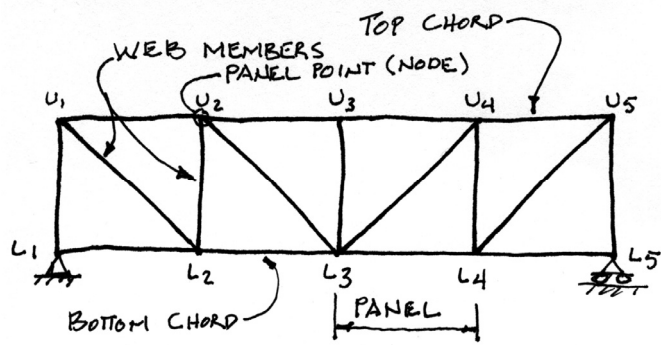
- Segments: left to right

## Joints

- Upper: U1, U2, U3...
- Lower: L1, L2, L3...

## Members

- Chords
- Web



# Force Systems

## 2D Trusses

- Concurrent Coplanar

## 3D Trusses

- Concurrent Non-Coplanar



Foster Bridge, 1889  
Ann Arbor, Michigan



University of Michigan Architectural Research Lab  
Unistrut System, Charles W. Attwood

# Stability and Determinacy

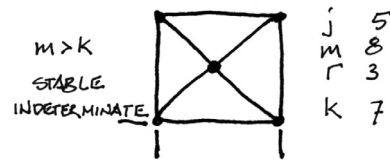
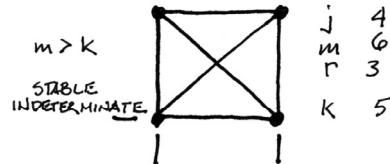
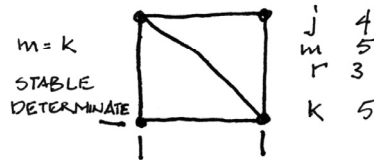
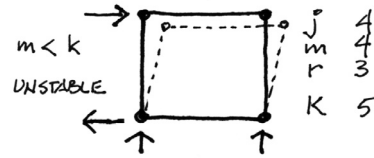
For:

- $j$  joints
- $m$  members
- $r$  reactions (restraints)

$$k = 2j - r$$

## Three conditions

- $m < k$  unstable
- $m = k$  stable and determinate
- $m > k$  stable and indeterminate



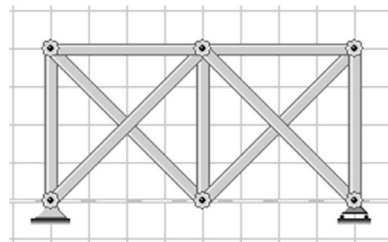
## Quiz

For each of the following trusses, determine whether they are:

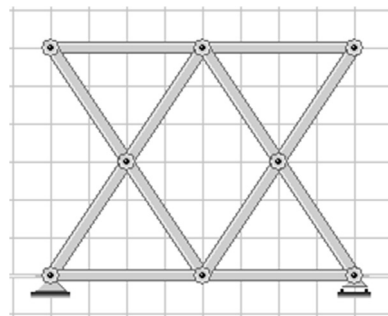
- A) Stable
- B) Unstable

$$k = 2j - r$$

- $m < k$  unstable
- $m = k$  stable and determinate
- $m > k$  stable and indeterminate



Truss 1



Truss 2

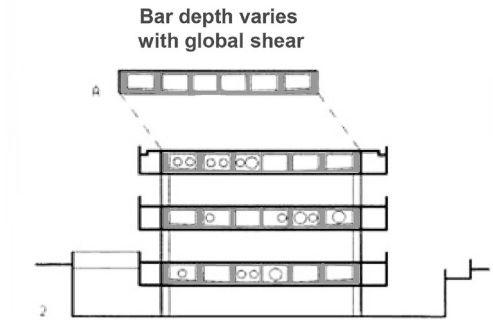
# Vierendeel "Truss"

Not a true truss

- Moment frame structure
- Rigid joints as moment connections
- Flexure in members



Vierendeel bridge at Grammene, Belgium  
Photo by Karel Roose



Salk Institute, La Jolla.  
Architect: Louis Kahn  
Engineer: Komendant and Dubin

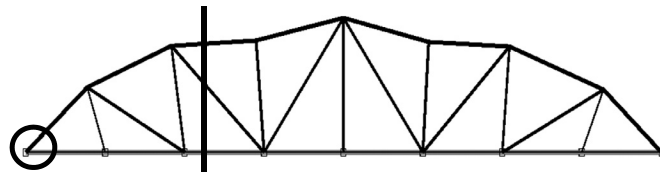
# Analysis

Method of Joints

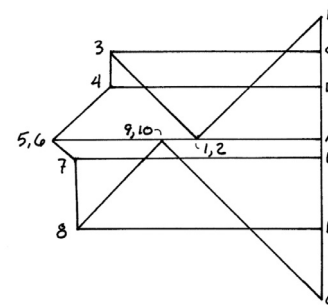
Method of Sections

Graphic Methods

- James Clerk Maxwell 1869
- M. Williot 1877
- Otto Mohr 1887
- Heinrich Müller-Breslau 1904

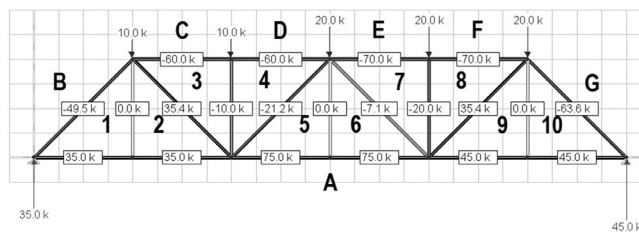


James Clerk Maxwell



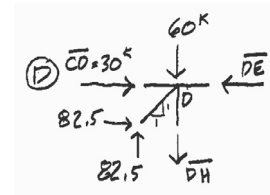
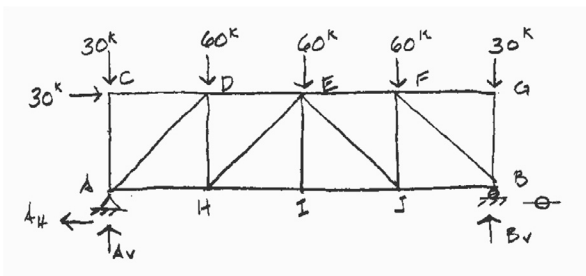
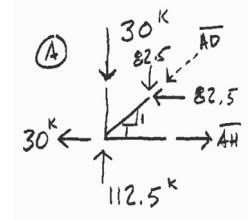
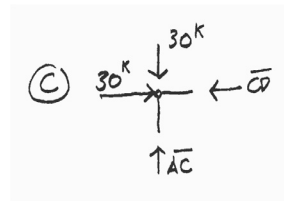
Computer Programs

- Dr. Frame (2D)
- STAAD Pro (2D or 3D)
- West Point Bridge Designer



## Method of Joints – procedure

1. Solve reactions (all external forces)
2. Inspect for zero force members (T's & L's)
3. Cut FBD of one joint
4. Show forces as orthogonal components
5. Solve with  $\Sigma F_H$  and  $\Sigma F_V$  (no  $\Sigma M$ )
6. Find resultant member forces (Pythagorean Formula)



Structures I

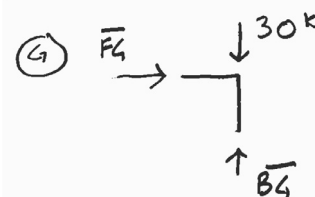
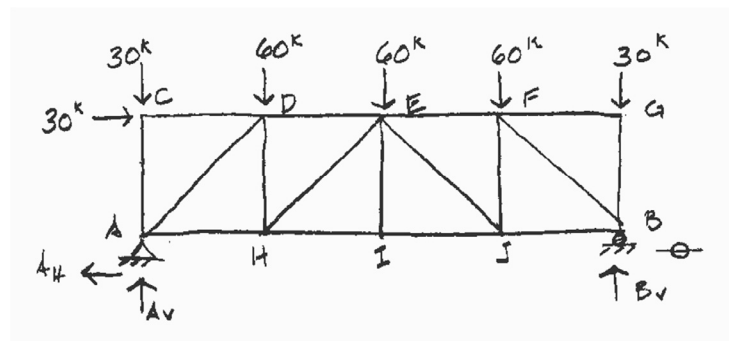
University of Michigan, Taubman College

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## Inspection of Zero Force Members

T – joints

L – joints



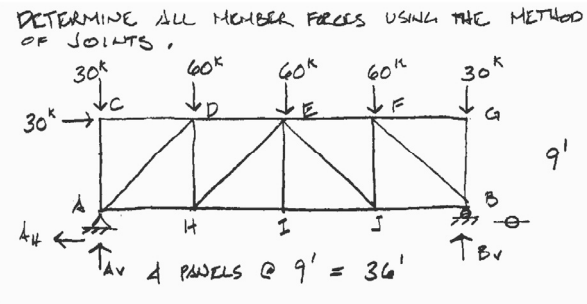
Structures I

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# Method of Joints - example

1. Solve the external reactions for the whole truss.



REACTIONS :

$$\sum F_H = 0 = 30 - A_H \quad \underline{A_H = 30^k \leftarrow}$$

$$\sum M @ A = 0 = 30(9) + 60(9) + 60(18) + 60(27) + 30(36) - B_V(36)$$

$$B_V(36) = 4590$$

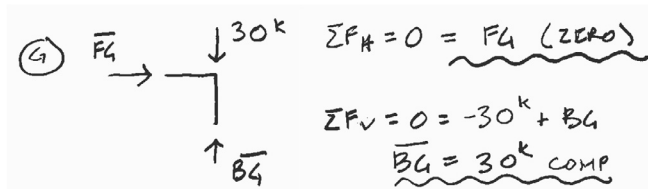
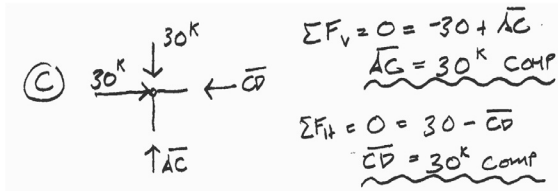
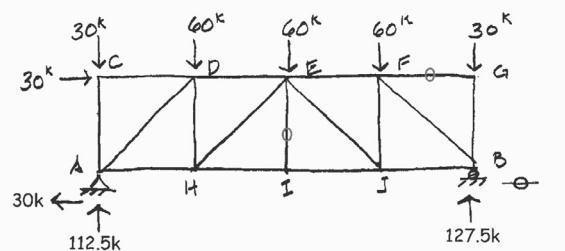
$$\underline{B_V = 127.5^k \uparrow}$$

$$\sum F_V = 0 = A_V + 127.5 - 2(30) - 3(60) = 0$$

$$\underline{A_V = 112.5^k \uparrow}$$

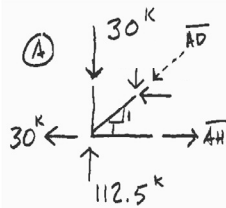
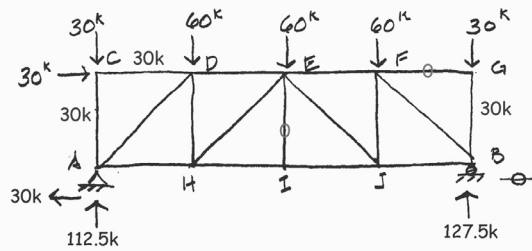
# Method of Joints - example

2. T or L joints by inspection.
3. Cut FBD of joint
4. Show orthogonal components
5. Solve by  $\sum F$  horz. and vert.



## Method of Joints - example

Continue with joints having only one unknown in either horizontal or vertical direction. Generally work starting at the reactions.



$$\Sigma F_v = -30 + 112.5 - \overline{AD}_v = 0$$

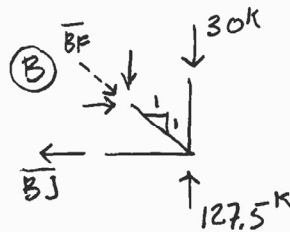
$$\overline{AD}_v = 82.5^k$$

$$\overline{AD}_h = \overline{AD}_v = 82.5^k$$

$$\overline{AD} = \sqrt{82.5^2 + 82.5^2} = 116.67^k \text{ COMP}$$

$$\Sigma F_h = 0 = -30 - 82.5 + \overline{AH}$$

$$\overline{AH} = 112.5^k \text{ TENSION}$$



$$\Sigma F_v = 0 = -30 + 127.5 - \overline{BF}_v$$

$$\overline{BF}_v = 97.5^k$$

$$\overline{BF}_h = \overline{BF}_v = 97.5^k$$

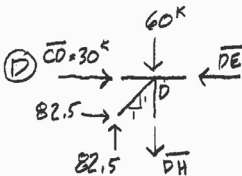
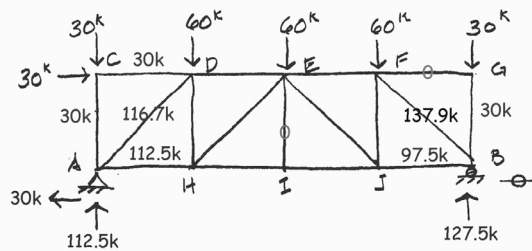
$$\overline{BF} = \sqrt{97.5^2 + 97.5^2} = 137.9^k \text{ COMP}$$

$$\Sigma F_h = 0 = 97.5 - \overline{BJ}$$

$$\overline{BJ} = 97.5^k \text{ TENS}$$

## Method of Joints - example

Continue moving across the truss, joint by joint. Solve by  $\Sigma F_H$  and  $\Sigma F_V$ .

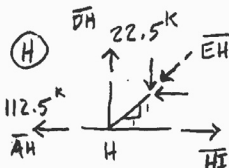


$$\Sigma F_h = 30 + 82.5 - \overline{DE} = 0$$

$$\overline{DE} = 112.5^k \text{ COMP}$$

$$\Sigma F_v = 82.5 - 60 - \overline{DH} = 0$$

$$\overline{DH} = 22.5^k \text{ TENSION}$$



$$\Sigma F_v = 0 = 22.5 - \overline{EH}_v$$

$$\overline{EH}_v = 22.5$$

$$\overline{EH}_h = \overline{EH}_v = 22.5$$

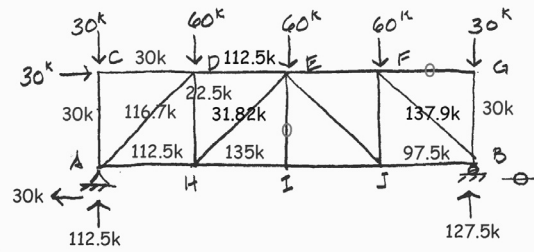
$$\overline{EH} = \sqrt{22.5^2 + 22.5^2} = 31.82^k \text{ COMP}$$

$$\Sigma F_h = 0 = -112.5 - 22.5 + \overline{HI}$$

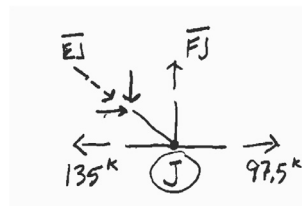
$$\overline{HI} = 135^k \text{ TENSION}$$

## Method of Joints - example

Continue moving across the truss, joint by joint. Choose joints that have only one unknown in each direction, horizontal or vertical.



$\sum F_H = 0 = -135 + \overline{IJ}$   
 $\overline{IJ} = 135^k$  TENSION  
 $\sum F_V = 0 = \overline{EI}$  (ZERO)

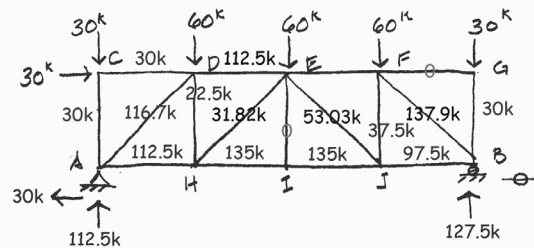


$\sum F_H = -135 + 97.5 + \overline{EJ}_H$   
 $\overline{EJ}_H = 37.5^k$   
 $\overline{EJ}_H = \overline{EJ}_V = 37.5^k$   
 $\overline{EJ} = \sqrt{37.5^2 + 37.5^2} = 53.03^k$  COMP  
 $\sum F_V = 0 = \overline{FJ} - 37.5 = 0$   
 $\overline{FJ} = 37.5^k$  TENSION

## Method of Joints - example

Solve the joints with the most members last.

Check that all forces balance.

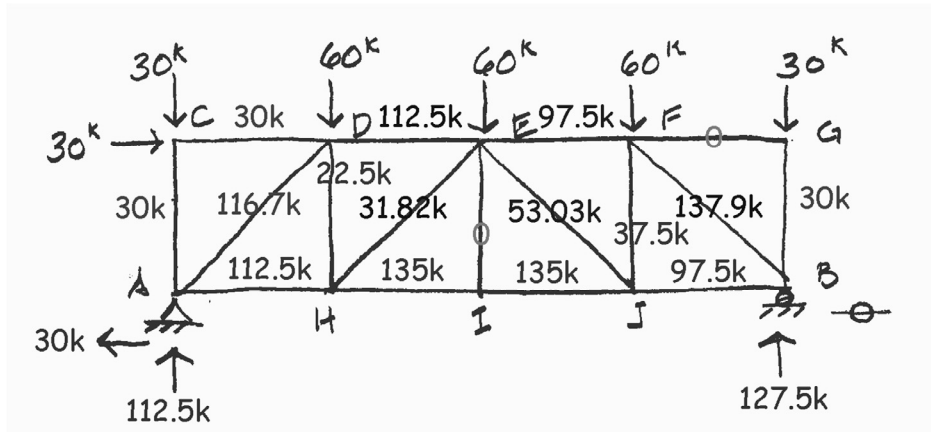


$\sum F_H = 0 = 112.5 + 22.5 - 37.5 - \overline{EF}$   
 $\overline{EF} = 97.5^k$  COMP  
 CHECK  
 $\sum F_V = 0 = -60 + 22.5 + 37.5 = 0$



# Method of Joints - example

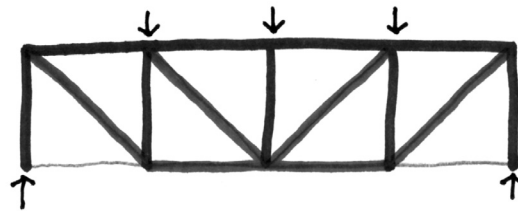
Inspect the final solution to see that it seems to make sense.



## Qualitative T or C

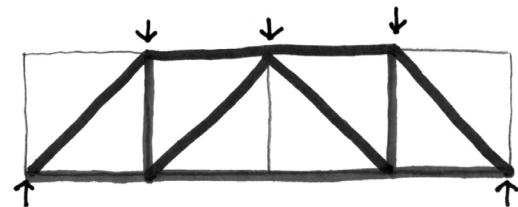
For typical gravity loading:  
(tension=red compression=blue)

Top chords are in compression

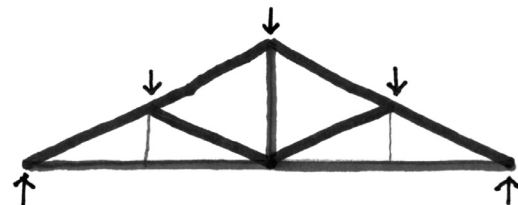


Bottom chords are in tension

Diagonals down toward center are in tension (usually)



Diagonals up toward center are in compression (usually)



# Qualitative Force

For spanning trusses with uniform loading:  
 (tension=blue compression=red)

Top and bottom chords greatest at center when flat (at maximum curvature or moment)

Diagonals greatest at ends (near reactions, i.e. greatest shear)

