Architecture 314

Structures I

Stress and Strain

- Stress
- Strain
- Analysis – ASD vs. LRFD
- Modes of Failure

$$\sigma = \frac{P}{A} \quad \frac{\text{FORCE}}{\text{AREA}}$$

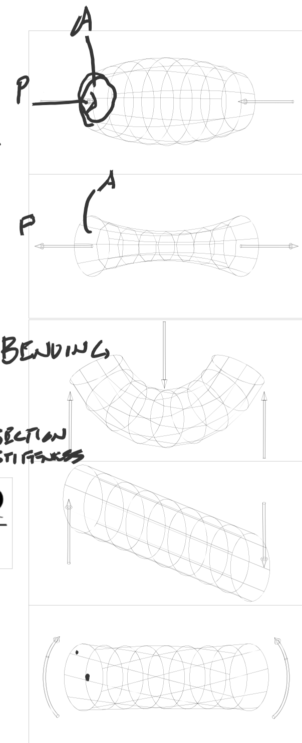
$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{M\,c}{I} \quad \text{BENDING}$$

I = SECTION STIFFNESS

$$\tau = \frac{P}{A} \; or \; \frac{VQ}{Ib}$$

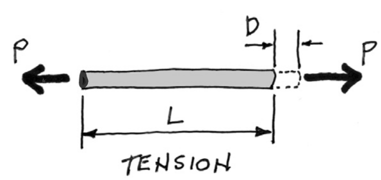
$$\tau = \frac{T\;r}{J}$$

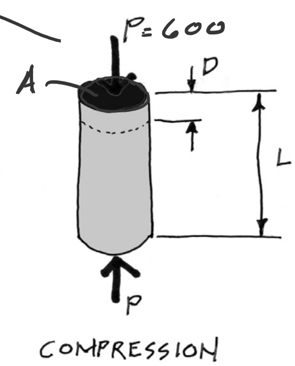


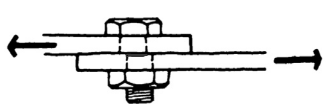
Stress

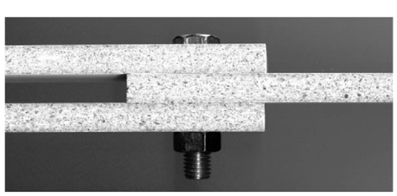
Stress is the result of a force being applied to the area of a material.

$$\sigma = \frac{P}{A} \quad \frac{\text{FORCE}}{\text{AREA}}$$







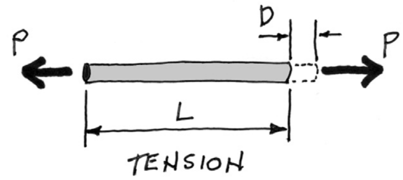


Shear Stress

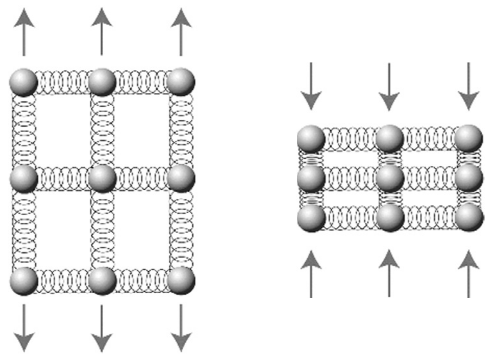
Strain

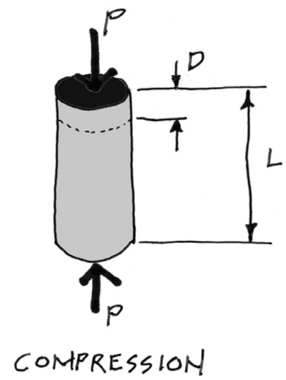
Strain is the amount of deformation in the material, per unit length.

$$\varepsilon = \frac{D}{L} \quad \frac{in}{in}$$



Deformation occurs either in stretching (tension) or in compressing (compression) but not always at the same rate.

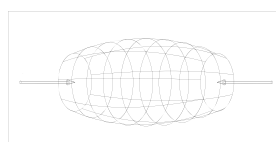




Types of Stress

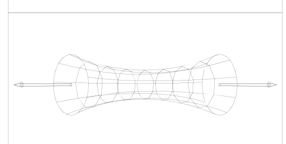
- Compression

$$\sigma = \frac{P}{A}$$



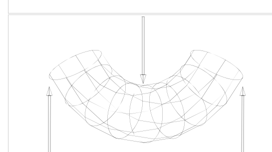
- Tension

$$\sigma = \frac{P}{A}$$



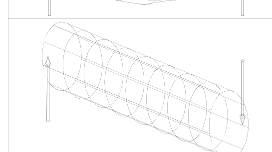
- Flexure

$$\sigma = \frac{M\ c}{I}$$



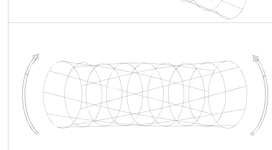
- Shear

$$\tau = \frac{P}{A} or \frac{VQ}{Ib}$$



- Torsion

$$\tau = \frac{T\ r}{J}$$



Stress Analysis

Allowable Stress Design (ASD)

- use applied loads (no F.S. on loads)
- reduce stress by a Factor of Safety F.S.

$$f_{actual} \le F_{allowable}$$

ACTUAL

$$f_{actual} = \frac{P}{A}$$

MATERIAL

$$F_{allowable} = F.S. \cdot f_{yield}$$

Load & Resistance Factored Design (LRFD)

- Use loads with safety factor γ
- Use factor on nominal strength ϕ

$$P_{load} \le P_{resisting}$$

$$P_{load} = \gamma \cdot P_{applied}$$

$$P_{resisting} = \phi \cdot P_{material}$$

Stress Calculations - example

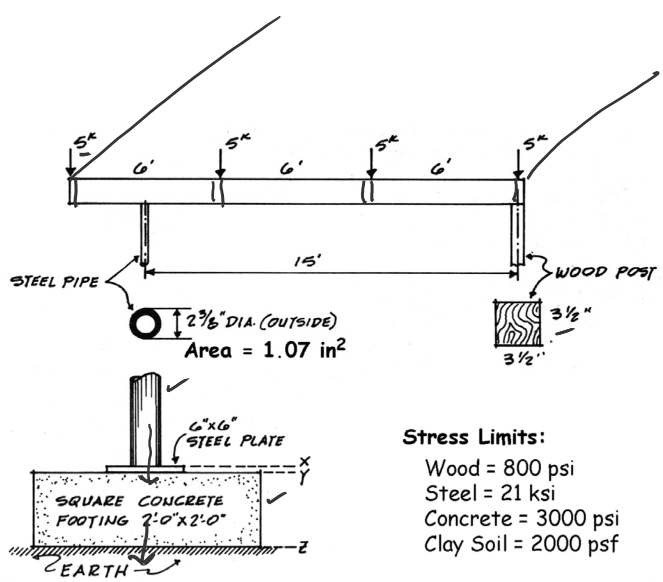
Find the stress in each material:

- wood
- steel
- concrete
- soil

Axial Compression

The stress equals the force spread over an area.

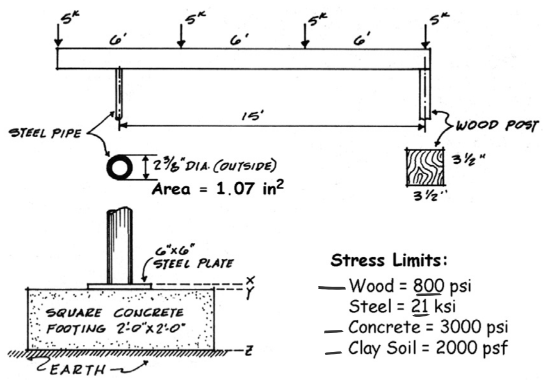
$$\sigma = \frac{P}{A}$$



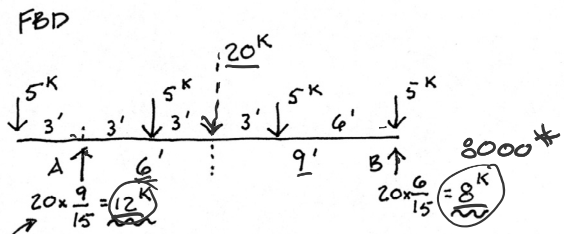
Stress Calculations

Find the force on the members

FBD to find the end reactions



Stress Limits:
- Wood = 800 psi
- Steel = 21 ksi
- Concrete = 3000 psi
- Clay Soil = 2000 psf



END REACTIONS BY PROPORTIONS

OR

ΣM AT EACH REACTION

$\Sigma M @ A = -5^K(3') + 5^K(3') + 5^K(9') + 5^K(15') - B(15')$

$B = 8^K$

$\Sigma F_v = 0 = -20 + A + 8 = 0$

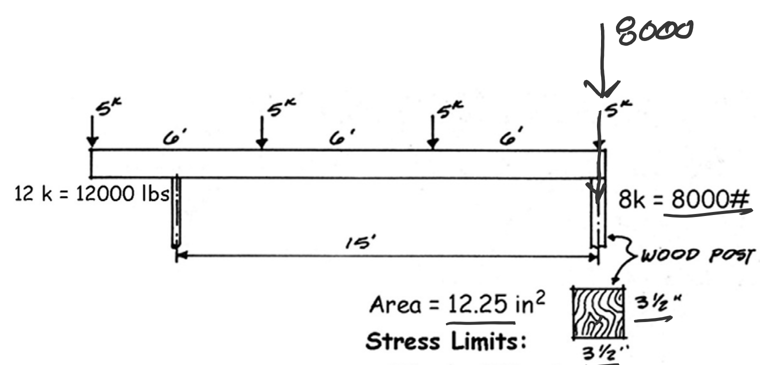
$A = 12^K$

Stress Calculations

for the right side (wood)

The stress equals the force on the member, spread over the sectional area of the member.

$$\sigma = \frac{P}{A}$$



Stress Limits:
Wood = 800 psi

$f = P/A$

Stress in Wood:

f = P/A

f = 8000lbs/12.25 in²

f = 653 psi

Allow. F = 800 psi

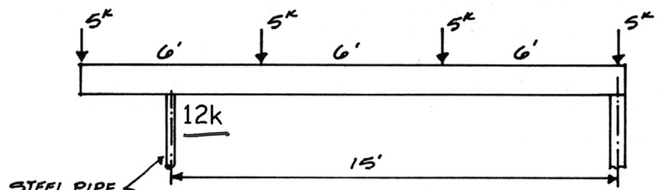
f < F ok ✓

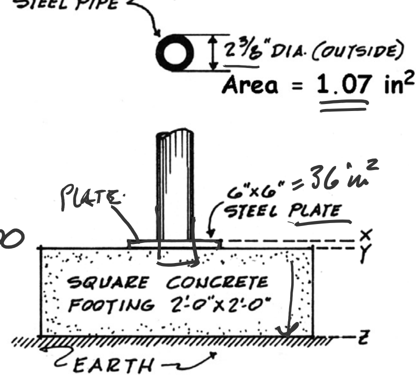
Stress Calculations

for the left side (steel pipe)

The stress equals the force spread over the area.

$$\sigma = \frac{P}{A}$$





TRY WOOD

$A = 12.25$

$\frac{P}{A}\ \frac{12000}{12.25} = 980 > 800$

X FAILS

Stress Limits:

Steel = 21 ksi

Concrete = 3000 psi   3 KSI

11.2 > 3 X

Stress in Steel:

f = P/A = 12k / 1.07 in²

f = 11.2 ksi < F = 21 ksi ok

Stress in Concrete:

f = 12000 lbs / 1.07 in² X

f = 11200 psi > 3000 psi FAILS!

f = 12000 / 36 in² = 333 psi

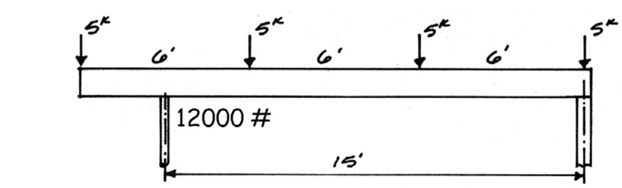
333 psi < 3000 psi ok

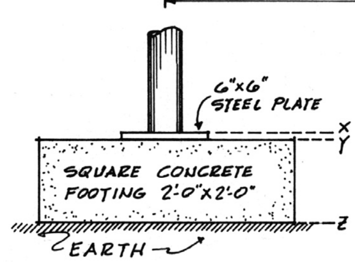
Stress Calculations

for the left side (foundation)

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$





Stress in Soil:

f = 12000 # / 4 sf

f = 3000 psf > 2000 psf FAILS! X

Stress Limits:

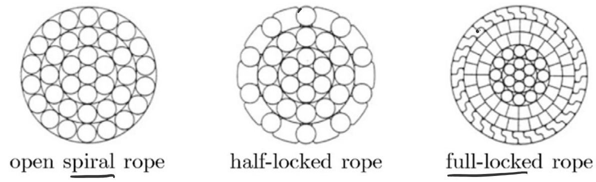
Clay Soil = 2000 psf

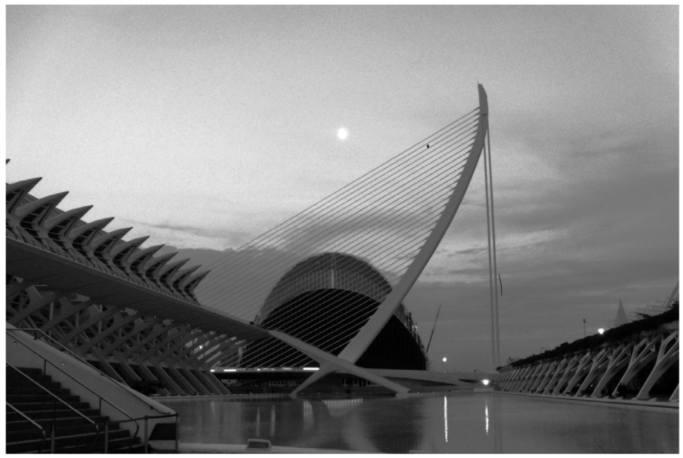
Stress Calculations

Axial Tension

The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$





Santiago Calatrava - Serreria Bridge - Valencia 2008

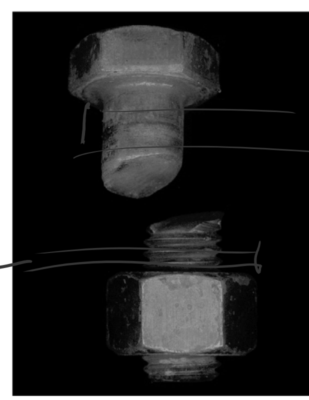
Stress Calculations

Shear

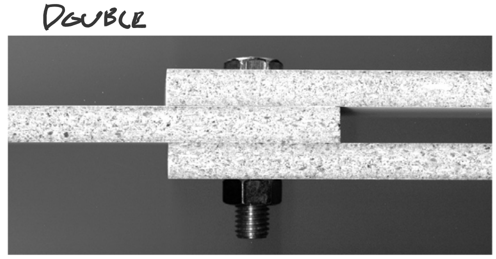
The stress equals the force spread over an area.

$$\sigma = \frac{P}{A}$$

SINGLE

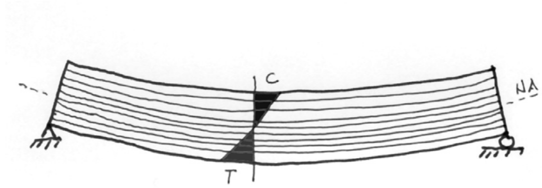


DOUBLE



Stress Calculations

Bending

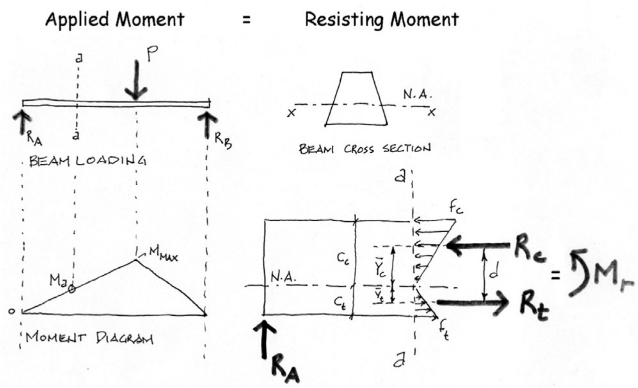


Flexure Stress

The stress is on the “fibers” or longitudinal layers

$$\sigma = \frac{M\ c}{\underline{I}}$$

MOMENT of INERTIA

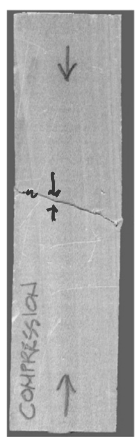


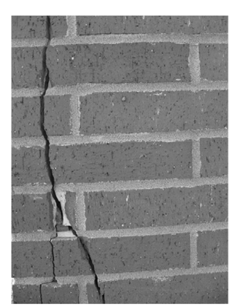
Shear Stress

The stress is between the longitudinal layers.

$$\tau = \frac{VQ}{Ib}$$

Modes of Failure





Strength P/A

- Tension rupture
- Compression crushing

Stability

- Column buckling
- Beam lateral torsional buckling

Serviceability

- Beam deflection
- Building story drift
- cracking