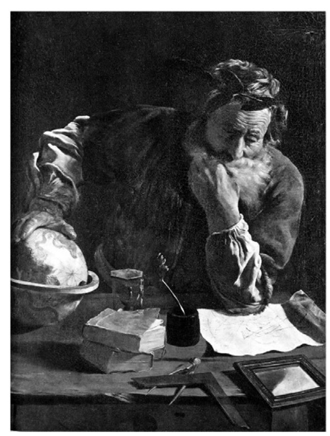
Equilibrium Equations: Two-Dimensional

- Archimedes' Lever
- Newton's First Law
- Loading Types
- End Conditions
- Free Body Diagrams
- End Reactions

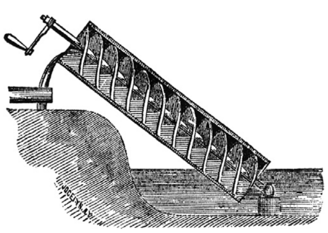
Archimedes of Syracuse (287 BC – 212 BC)

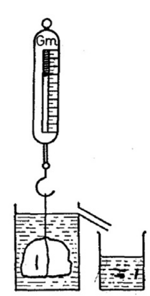
Greek mathematician, engineer, inventor

- The Lever (*On the Equilibrium of Planes*)
- The Screw (water pump)
- Greek Fire (to burn boats)
- Archimedes' Principle (density measure)
- Block and Tackle (for lifting on boats)
- Catapult
- Odometer
- Mathematical observations on circles and spheres



by Domenico-Fetti





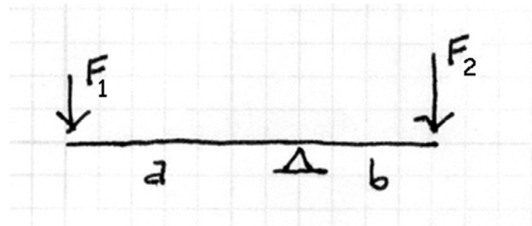
Archimedes' principle.

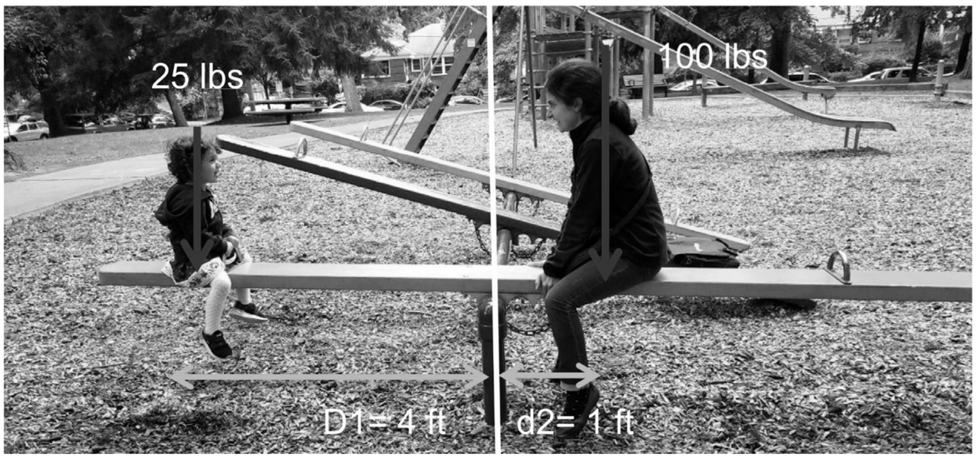
Archimedes Lever

Two forces will balance at distances reciprocally proportional to their magnitudes.

$$\boxed{\mathbf{F}_1 \times a = \mathbf{F}_2 \times b}$$

$$\boxed{\mathbf{F}_1 = \mathbf{F}_2 \frac{b}{a}}$$



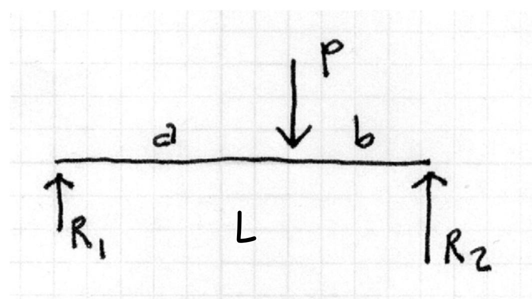


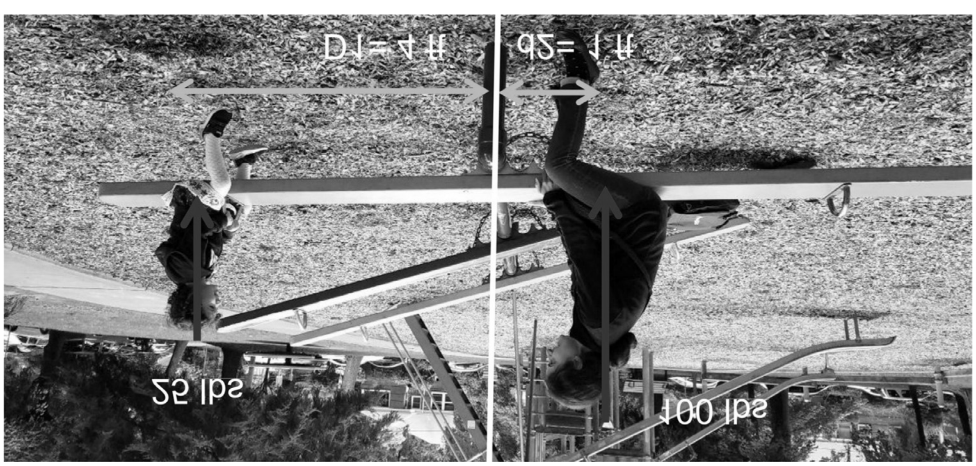
Archimedes Lever

Two forces will balance at distances reciprocally proportional to their magnitudes.

Applied to beam end reactions:

$$\boxed{\mathbf{R}_1 = \mathbf{P} \frac{b}{L}} \qquad \boxed{\mathbf{R}_2 = \mathbf{P} \frac{a}{L}}$$

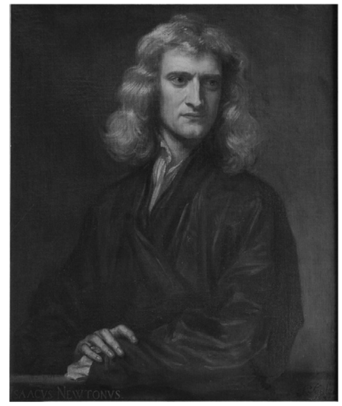




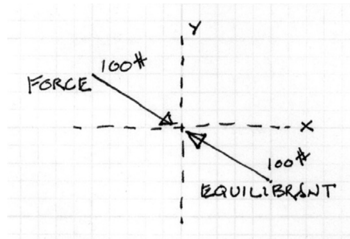
Newton's First Law

An object at rest will remain at rest unless acted upon by an outside, external net force.

$$\boxed{\sum \mathbf{F}_x = 0} \quad \boxed{\sum \mathbf{F}_y = 0} \quad \boxed{\sum \mathbf{M} = 0}$$



Sir Isaac Newton 1643 - 1726



Horizontal Equilibrium

$$\boxed{\sum \mathbf{F}_x = 0}$$

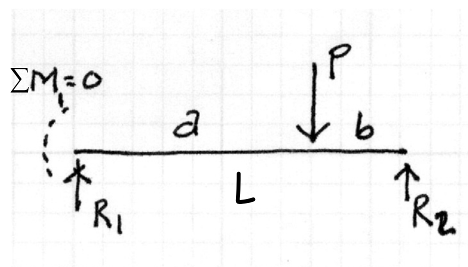
Vertical Equilibrium

$$\boxed{\sum \mathbf{F}_y = 0 = \mathbf{R}_1 + \mathbf{R}_2 - \mathbf{P}}$$

$$\boxed{\mathbf{R}_1 + \mathbf{R}_2 = \mathbf{P}}$$

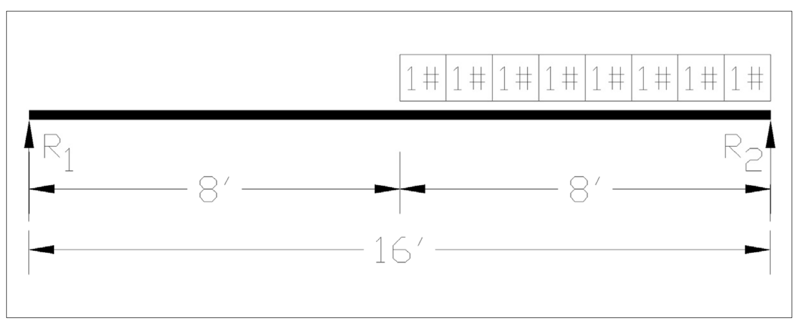
Rotational Equilibrium

$$\boxed{\sum \mathbf{M}_1 = 0 = \mathbf{P}a - \mathbf{R}_2\mathbf{L}} \quad \boxed{\mathbf{R}_2 = \frac{\mathbf{P}a}{\mathbf{L}}}$$



Quiz

Find the end reactions R1 and R2



Support Conditions

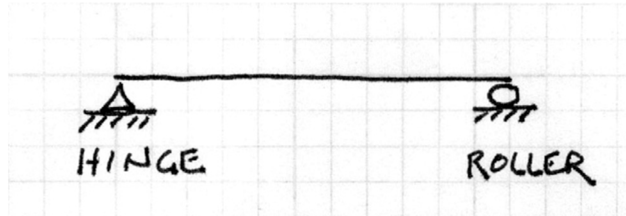
Roller

Fixed in Fy

Hinge

Fixed in Fx

Fixed in Fy

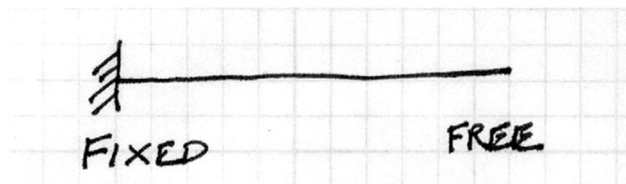


Fixed

Fixed in Fx

Fixed in Fy

Fixed in Mz



Free

Free in Fx

Free in Fy

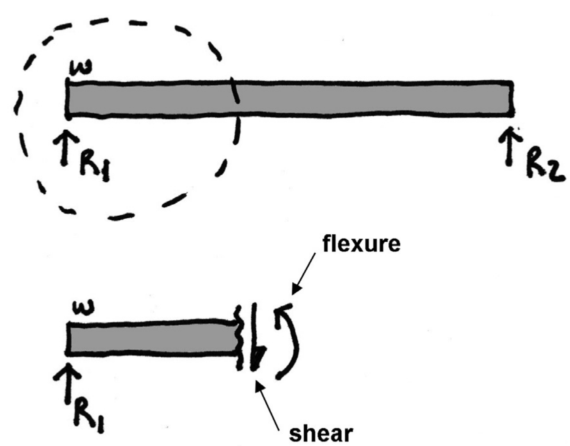
Free in Mz

Free Body Diagrams

A Free Body Diagram (FBD) is a part cut from a larger force system.

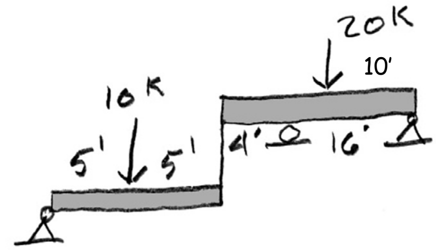
When the FBD is cut free, all "exposed" forces are shown

If the complete system is in static equilibrium, then the FBD with forces at the cut will also be in equilibrium

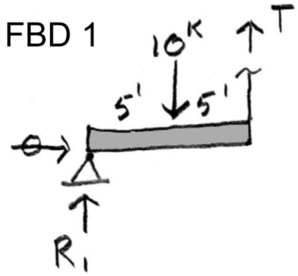


Free Body Diagrams

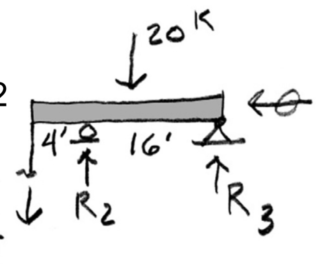
A Free Body Diagram (FBD) can be used as a step in solving the external forces



FBD 1



FBD 2



End Reactions

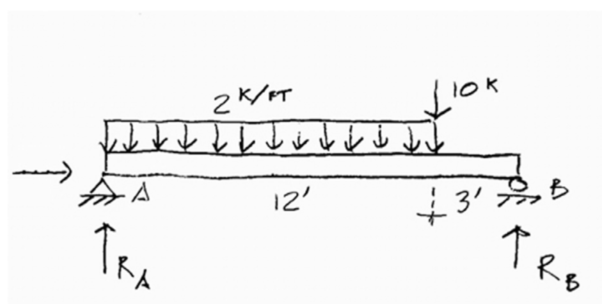
Example 1

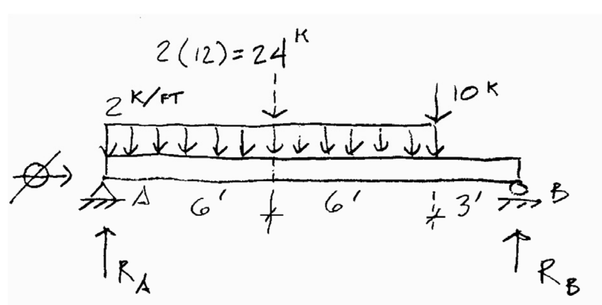
Given: Loads, Supports, Dimensions

1. Label components of reactions. Depending on the support condition, include vertical, horizontal and rotational.

2. Convert area loads to point loads through the centroid (balance point) of the area.

3. Since there is only one horizontal force, it must equal zero.





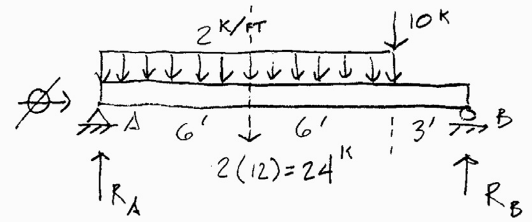
End Reactions

Example 1

4. Use the summation of moments about A to find R_B.

5. Use the summation of moments about B to find R_A.

6. Check calculation by summing vertical forces.



$$\Sigma M@A = 0 = 24(6) + 10(12) - R_B(15)$$
$$R_B(15) = 264$$
$$R_B = 17.6^K \uparrow$$

$$\Sigma M@B = 0 = R_A(15) - 24(9) - 10(3)$$
$$R_A(15) = 246$$
$$R_A = 16.4^K \uparrow$$

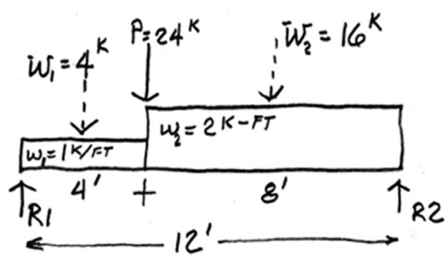
Check
$$\Sigma F_V = 0 = 17.6 - 24 - 10 + 16.4 = 0 \checkmark$$

End Reactions

Example 2

Given: Loads, Supports, Dimensions

1. Use the summation of moments about R_2 to find R_1.
2. Use the summation of moments about R_1 to find R_2.
3. Check calculation by summing vertical forces.



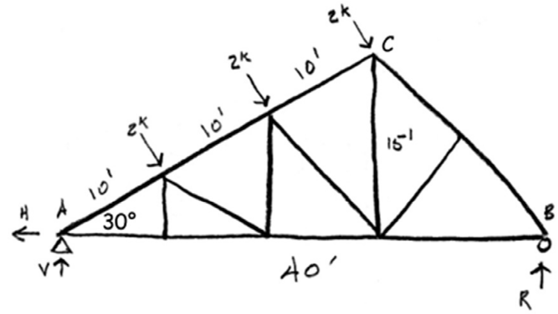
$$\Sigma M@R_2 = R1(12') - 4^k(10') - 24^k(8') - 16^k(4') = 0$$
$$R1(12') = 296^{k\text{-}ft}$$
$$R1 = 24.67^k$$

$$\Sigma M@R_1 = 4^k(2') + 24^k(4') + 16^k(8') - R2(12') = 0$$
$$R2(12') = 232^{k\text{-}'}$$
$$R2 = 19.33^k$$

$$\Sigma F_v = 24.67^k + 19.33^k - 4^k - 24^k - 16^k = 0$$
$$\Sigma F_v = 0 \checkmark \text{ok}$$

End Reactions   Example 3

1. Label components of reactions. You will need one equation for each unknown reaction.
2. Write an equation for the summation of moments.
3. Write an equation for the summation of vertical forces.
4. Write an equation for the summation of horizontal forces.
5. It is good practice to write one additional equation to check the results. In this case summation of moments at C also = 0.



$\Sigma M_A = 0$

$2(10) + 2(20) + 2(30) - R(40) = 0$

$R(40) = 120$

$R = 3^k \uparrow$

$\Sigma F_V = 0$

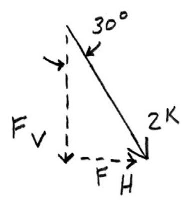
$-1.732 - 1.732 - 1.732 + 3 + V = 0$

$V = 2.196^k \uparrow$

$\Sigma F_H = 0$

$1 + 1 + 1 - H = 0$

$H = 3^k \leftarrow$



$F_V = \cos 30°(2) = 1.732^k \downarrow$

$F_H = \sin 30°(2) = 1.0^k \rightarrow$