

## Equilibrium Equations: Two-Dimensional

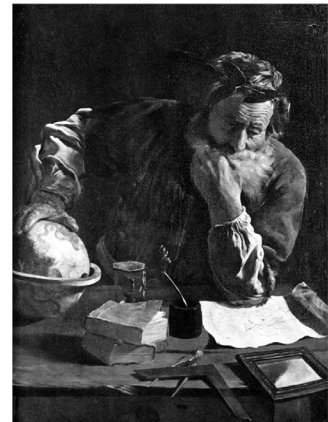
- Archimedes' Lever
- Newton's First Law
- Loading Types
- End Conditions
- Free Body Diagrams
- End Reactions



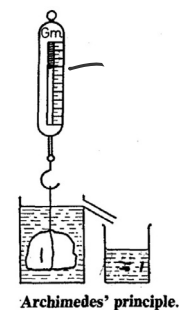
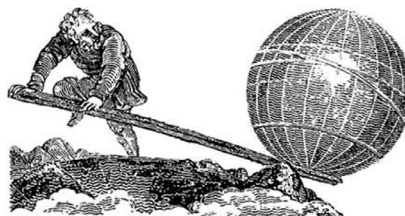
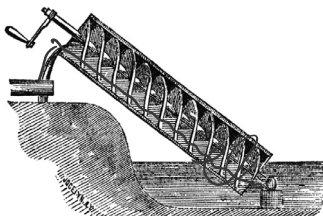
## Archimedes of Syracuse (287 BC – 212 BC)

Greek mathematician, engineer, inventor

- The Lever (*On the Equilibrium of Planes*)
- The Screw (water pump)
- Greek Fire (to burn boats)
- Archimedes' Principle (density measure)
- Block and Tackle (for lifting on boats) ✓
- Catapult ✓
- Odometer ✓
- Mathematical observations on circles and spheres



by Domenico-Fetti

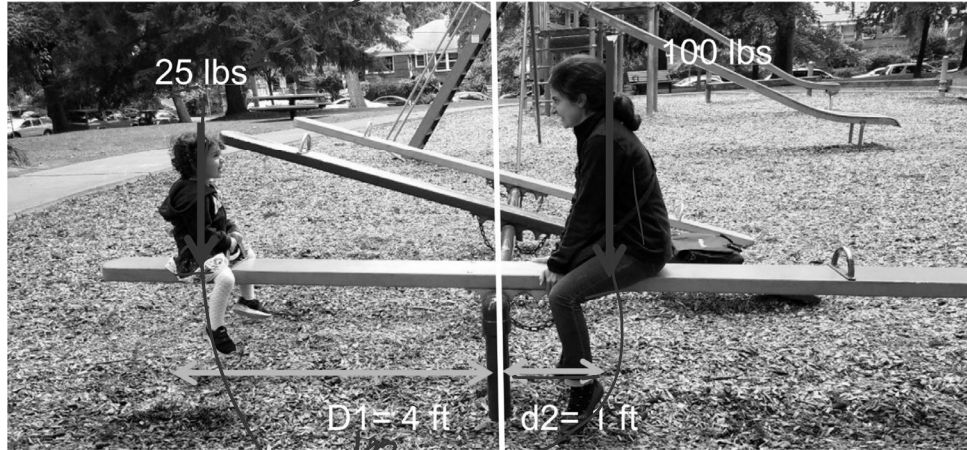
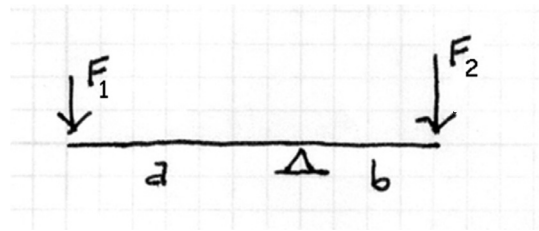


# Archimedes Lever

Two forces will balance at distances reciprocally proportional to their magnitudes.

$$F_1 \times a = F_2 \times b$$

$$F_1 = F_2 \frac{b}{a} \quad \frac{1}{4} = 25 \text{ lbs}$$



# Archimedes Lever

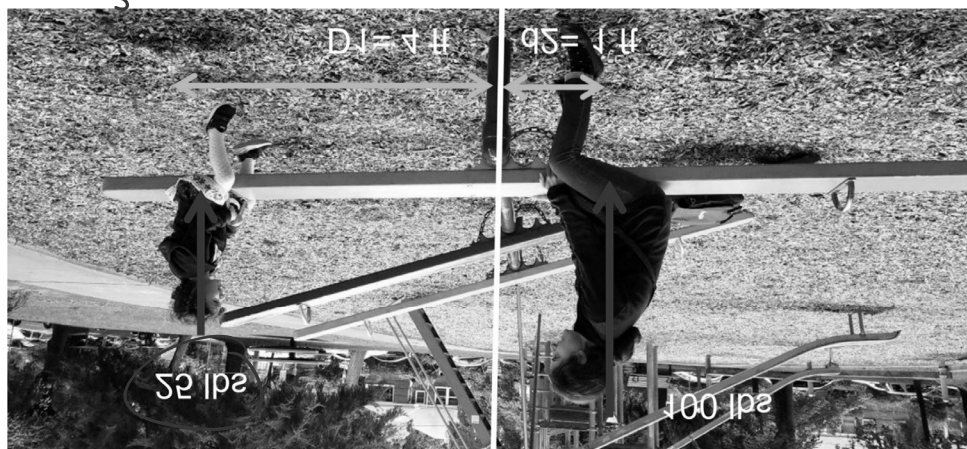
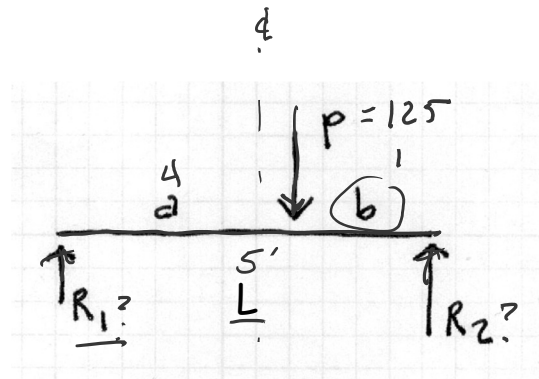
Two forces will balance at distances reciprocally proportional to their magnitudes.

Applied to beam end reactions:

$$R_1 = P \frac{b}{L}$$

$$R_2 = P \frac{a}{L}$$

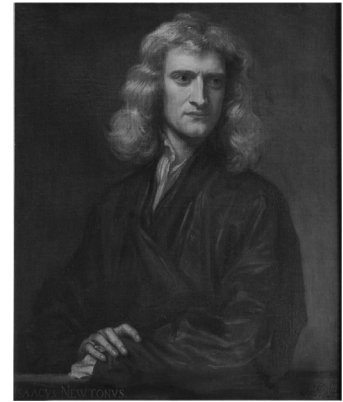
$$R_1 = 125 \frac{1}{5} = 25$$



# Newton's First Law

An object at rest will remain at rest unless acted upon by an outside, external net force.

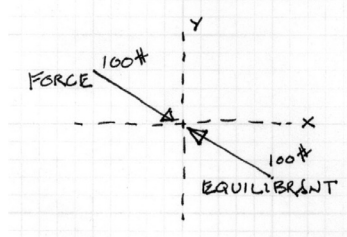
2 DIM  $\sum \underline{F}_x = 0$   $\sum \underline{F}_y = 0$   $\sum \underline{M} = 0$  3 RD



Sir Isaac Newton 1643 - 1726

Horizontal Equilibrium

$\underline{\underline{\sum F_x = 0}}$  ✓

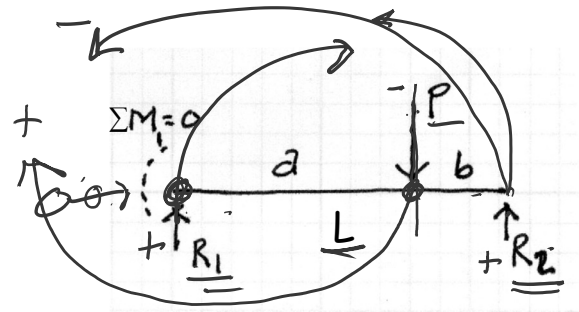


Vertical Equilibrium

$\underline{\underline{\sum F_y = 0 = R_1 + R_2 - P}}$   
 $\underline{\underline{R_1 + R_2 = P}}$  ✓

Rotational Equilibrium

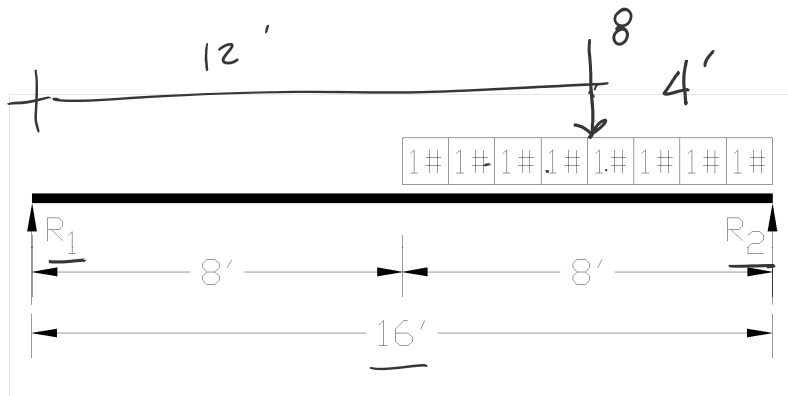
$\underline{\underline{\sum M_1 = 0 = Pa - R_2 L}}$   $\underline{\underline{R_2 = \frac{Pa}{L}}}$



## Quiz

Find the end reactions  
R1 and R2

$R_1 < R_2$



$R = P \frac{x}{L}$  ← 8 or 8

# Support Conditions

## Roller

Fixed in  $F_y$

## Hinge

Fixed in  $F_x$

Fixed in  $F_y$

## Fixed

Fixed in  $F_x$

Fixed in  $F_y$

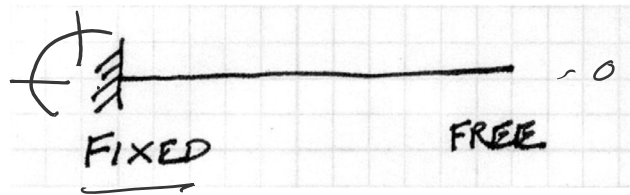
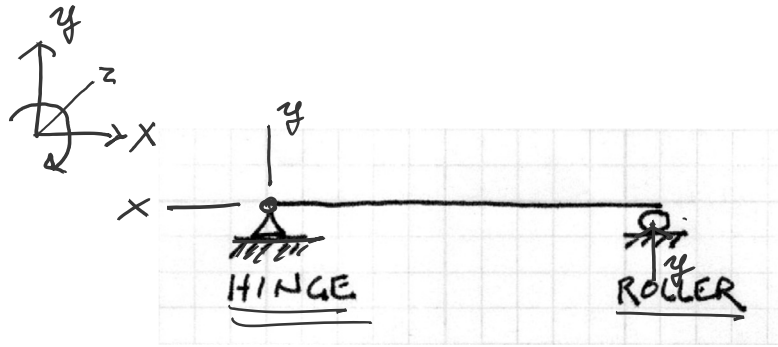
Fixed in  $M_z$

## Free

Free in  $F_x$

Free in  $F_y$

Free in  $M_z$

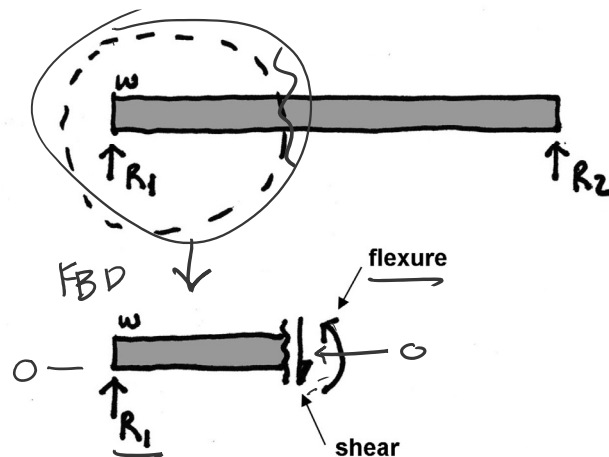


# Free Body Diagrams

A Free Body Diagram (FBD) is a part cut from a larger force system.

When the FBD is cut free, all "exposed" forces are shown

If the complete system is in static equilibrium, then the FBD with forces at the cut will also be in equilibrium



# Free Body Diagrams

A Free Body Diagram (FBD) can be used as a step in solving the external forces

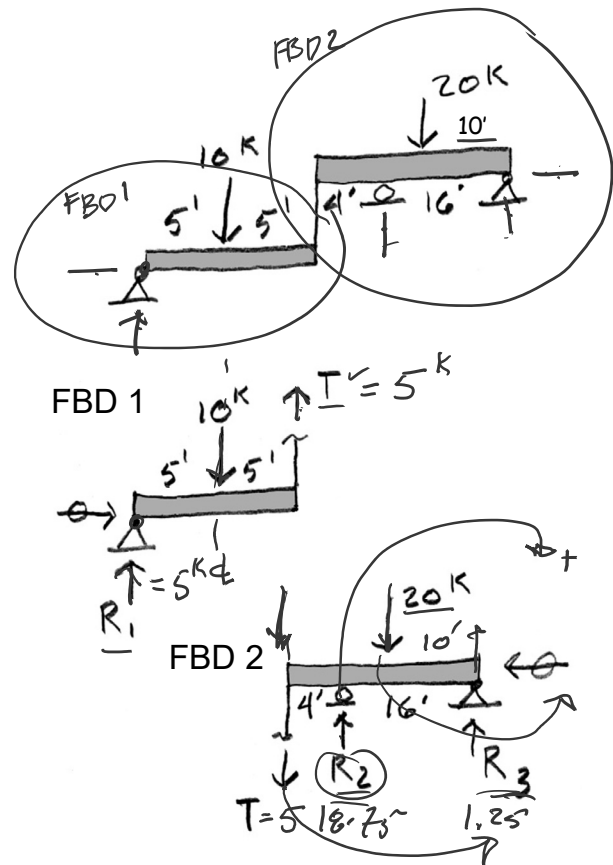
$$\sum F_v = R_3 - 20 + R_2 = 0$$

$$R_3 = 1.25$$

$$\sum M_{e_3} = -5(20) - 20(10) + R_2(16)$$

$$= -100 - 200 + R_2(16)$$

$$R_2 = \frac{300}{16} = 18.75$$

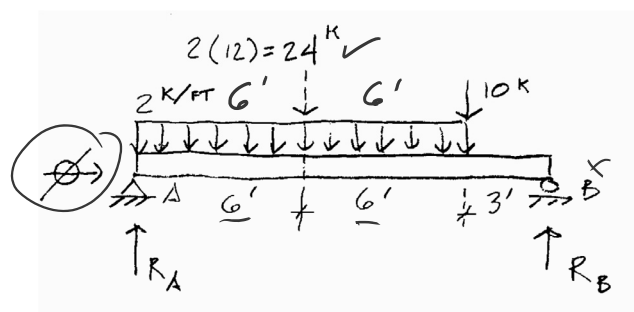
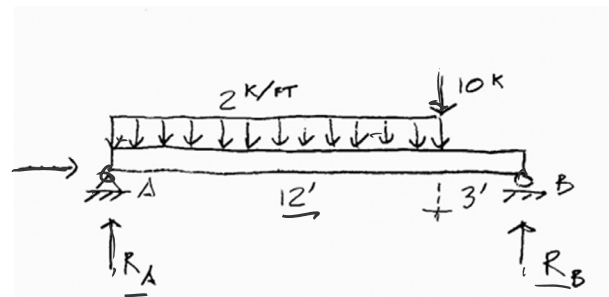


## End Reactions

### Example 1

Given: Loads, Supports, Dimensions

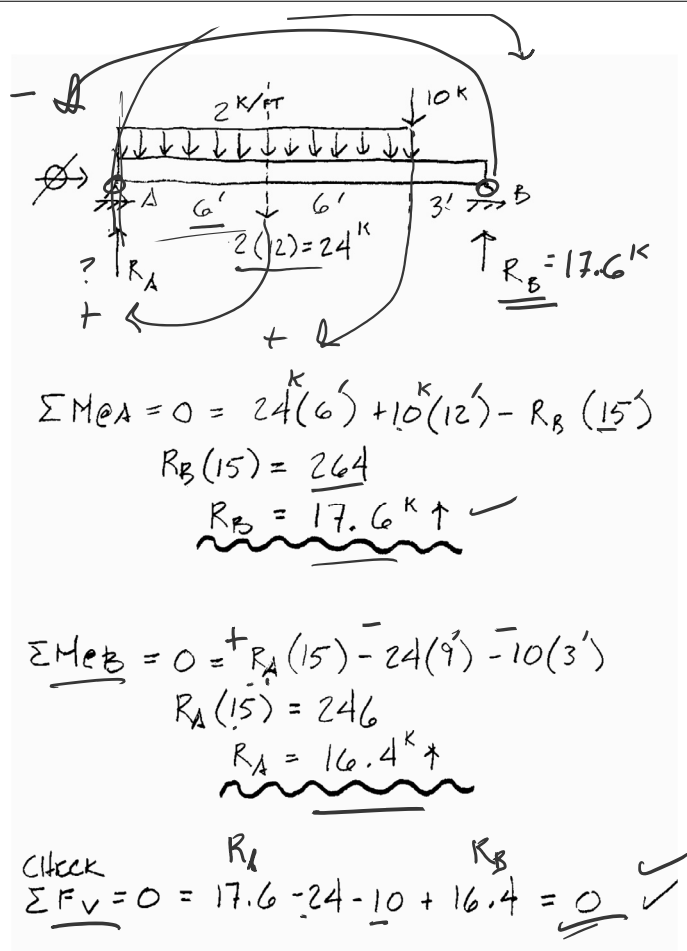
1. Label components of reactions. Depending on the support condition, include vertical, horizontal and rotational.
2. Convert area loads to point loads through the centroid (balance point) of the area.
3. Since there is only one horizontal force, it must equal zero.



## End Reactions

### Example 1

- Use the summation of moments about A to find  $R_B$ .
- Use the summation of moments about B to find  $R_A$ .
- Check calculation by summing vertical forces.

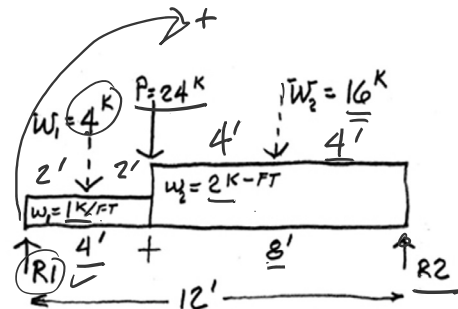


## End Reactions

### Example 2

Given: Loads, Supports, Dimensions

- Use the summation of moments about  $R_2$  to find  $R_1$ .
- Use the summation of moments about  $R_1$  to find  $R_2$ .
- Check calculation by summing vertical forces.



$$\sum M @ R_2 = (R_1)(12') - 4(10') - 24(8') - 16(4') = 0$$

$$R_1(12') = 296 \text{ k}\cdot\text{ft}$$

$$R_1 = 24.67 \text{ k}$$

$$\sum M @ R_1 = 4(2') + 24(4') + 16(8') - R_2(12') = 0$$

$$R_2(12') = 232 \text{ k}\cdot\text{ft}$$

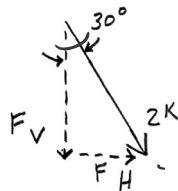
$$R_2 = 19.33 \text{ k} \quad \checkmark$$

$$\sum F_v = 24.67 + 19.33 - 4 - 24 - 16 = 0$$

$$\sum F_v = 0 \quad \checkmark \text{ OK}$$

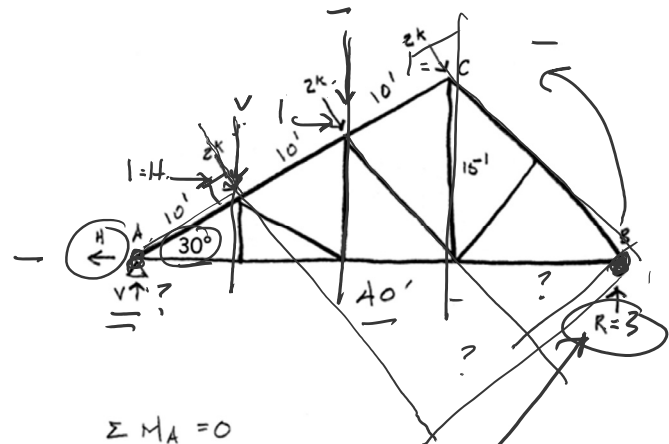
## End Reactions Example 3

1. Label components of reactions. You will need one equation for each unknown reaction.
2. Write an equation for the summation of moments.
3. Write an equation for the summation of vertical forces.
4. Write an equation for the summation of horizontal forces.
5. It is good practice to write one additional equation to check the results. In this case summation of moments at C also = 0.



$$F_v = \cos 30^\circ (2) = 1.732^k \downarrow$$

$$F_h = \sin 30^\circ (2) = 1.0^k \rightarrow$$



$$\sum M_A = 0$$

$$2(10) + 2(20) + 2(30) - R(40) = 0$$

$$R(40) = 120$$

$$R = 3^k \uparrow$$

$$\sum F_v = 0$$

$$-1.732 - 1.732 - 1.732 + 3 + V = 0$$

$$V = 2.196^k \uparrow$$

$$\sum F_h = 0$$

$$1 + 1 + 1 - H = 0$$

$$H = 3^k \leftarrow$$