Architecture 314

Structures I

Equilibrium of a Particle

Cable Systems

- Catenary Cable Systems
- Solving Cable Forces
- Cable Net Systems

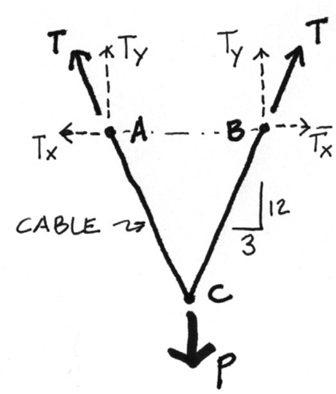
ILEK , Stuttgart

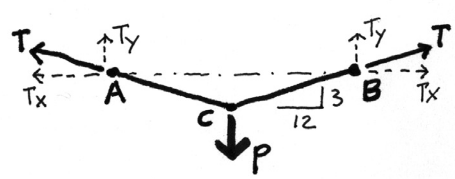
Cables

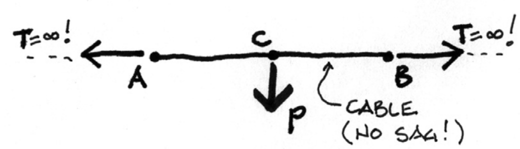
Both the sag and the load determine the force in the cable.

The less the sag, the higher the force in the cable.

The vertical component of the cable force in the case shown remains constant . Ty=P/2 But since the resultant follows the direction of the cable, Tx becomes greater as sag decreases. With no sag, Tx is infinite !



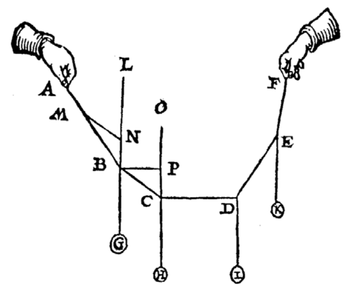


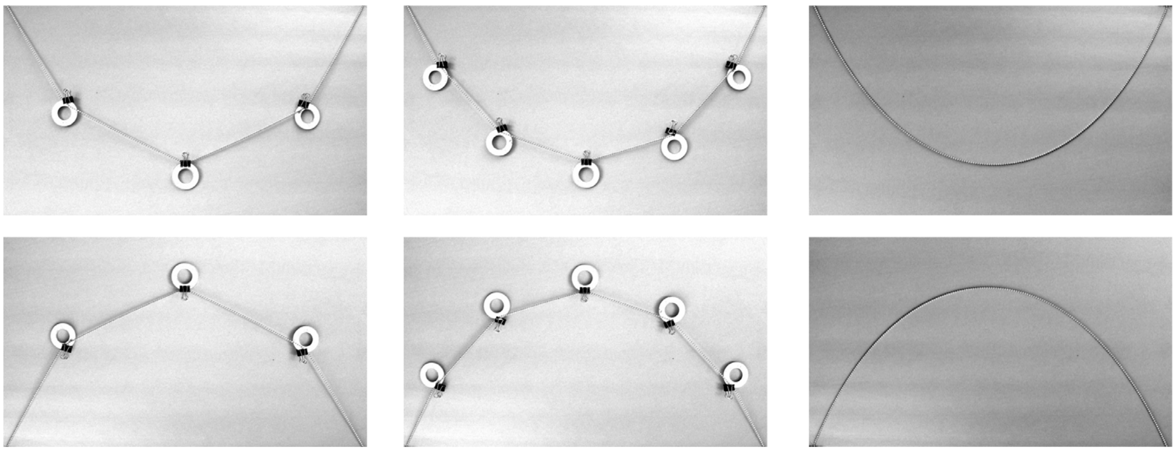


Catenary Shapes

The shape of the catenary depends on the loading. Simon Stevin showed this experimentally in 1585 with a weighted cord.

Because the cord has no resistance to moment, it assumes the shape (reversed) of the moment diagram for a beam with the same loading.





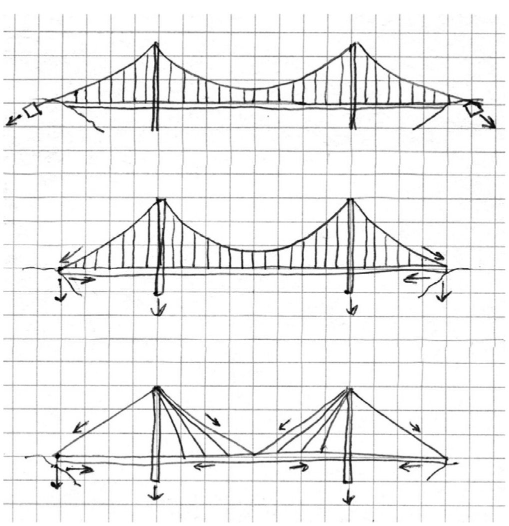
Cable Bridges

Long span bridges are often cable supported.

There are 3 common types:

- earth anchored cable
- deck anchored cable
- cable stayed

Gordie Howe International Bridge, Detroit
Photo: Bridging North America



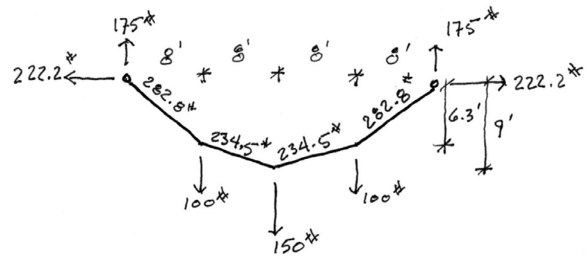
Golden Gate Bridge - center span: 1.28 mi total span: 1.7 mi

Solving Cables Forces

even supports – symmetric loading
Procedure:

1. Solve all external forces (reactions)
- If symmetric, then the reactions at each end are equal
- Use 3 equilibrium equations to solve

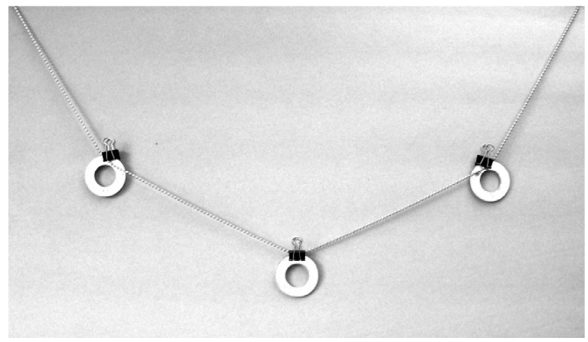
$$\sum Fx = 0 \quad \sum Fy = 0 \quad \sum M = 0$$



- The moment is also 0 at any point in the cable since the cable cannot support flexure,

2. Start at reactions and move inward
- Draw FBD of system from reaction inward
- Slope is proportional to forces
- Use force equations to solve at cut
- Find forces and slope of each section

$$\sum Fx = 0$$
$$\sum Fy = 0$$



Cables Forces

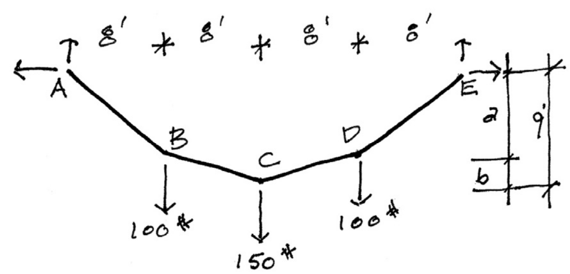
Example 1
even supports – symmetric loading

Systems with supports on the same level can be solved without simultaneous equations since the horizontal reactions pass through the same point.

If the system is symmetric then the end reactions will be equal, and can be solved by summing vertical forces.

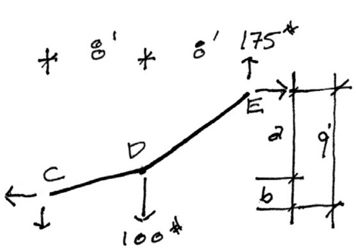
Using a FBD of cut section and summing moments at a point with know dimensions (in this case C) the horizontal forces can be found.

Full Diagram



$$\Sigma F_V = 0 = +A_Y - 100^{\#} - 150^{\#} - 100^{\#} + E_Y$$
$$A_Y + E_Y = 350^{\#}$$
$$A_Y = E_Y = 175^{\#}\uparrow$$

FBD 1



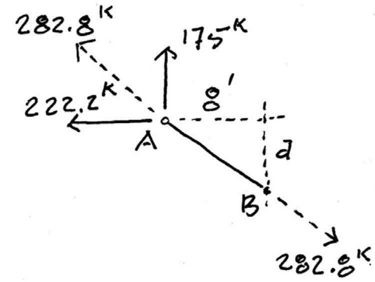
FBD 1

$$\Sigma M@C = 0 = 100^{\#}(8') - 175^{\#}(16') + E_x(9')$$
$$E_x(9') = 2000^{'\text{-}\#}$$
$$E_x = 222.2^{\#}$$
$$\Sigma F_H = 0 = -A_x + E_x = 0$$
$$A_x = E_x = 222.2^{\#}$$

Cables Forces

Example 1
even supports – symmetric loading

Cable slopes and forces can now be found by ratios and Pythagorean formula.

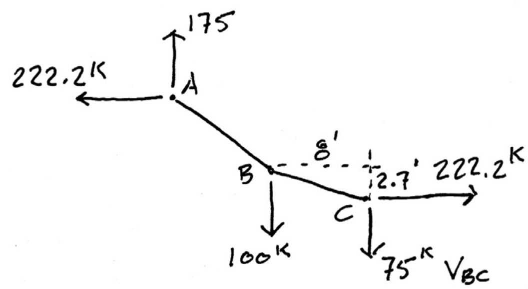


CABLE A-B AND D-E

$\frac{175}{222.2} : \frac{d}{8} \qquad d = 6.3$

CABLE FORCE $= \sqrt{175^2 + 222.2^2} = 282.8^{\#}$

FBD 2



CABLES B-C AND C-D

$9' - 6.3' = 2.7'$

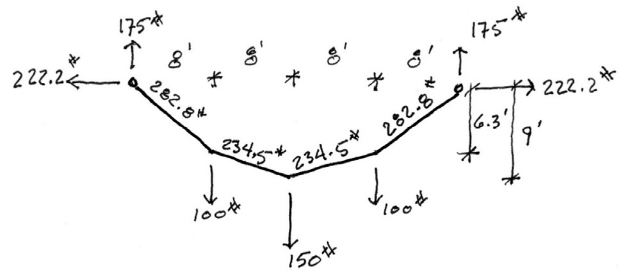
$\Sigma F_V = 0 = 175 - 100 - V$

$V_{BC} = 75^K$

CABLE FORCE $= \sqrt{75^2 + 222.2^2} = 234.5^K$

Cables Forces

Example 1
even supports – symmetric loading



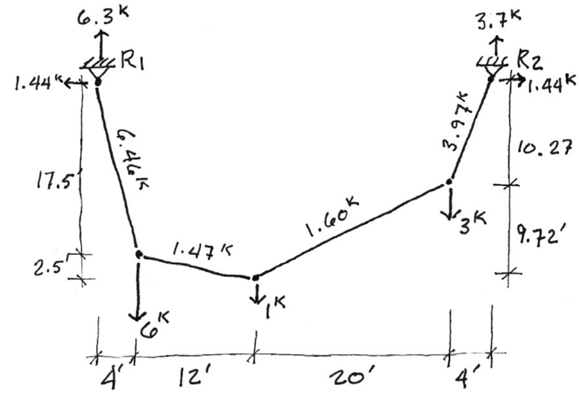
Cable slopes and forces can now be found by ratios and Pythagorean formula.

Note that the forces in each length are different.
This type of system should not be confused with cable systems using pulleys, where the force remains the same throughout the length of the cable.

Solving Cables Forces

even supports – asymmetric loading
Procedure:

1. Solve all external forces (reactions)
- If asymmetric, then the vertical reactions are not equal.
- With no applied horizontal loads, the horizontal reactions are equal.
- The vertical reactions can be solved either by summing moments at each end or by proportions as in a beam.
- The moment is also 0 at any point in the cable since the cable cannot support flexure,



2. Start at reactions and move inward
- Draw FBD of system from reaction inward
- Slope is proportional to forces
- Use force equations to solve internal cable force at each cut point
- Find forces and slope of each section

$$\sum Fx = 0$$
$$\sum Fy = 0$$

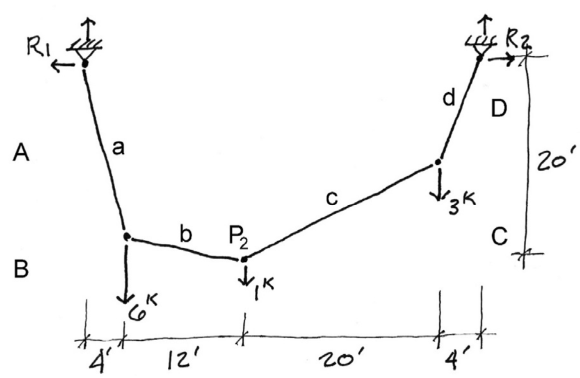
Cables Forces

Example 2
even supports – asymmetric loading

Systems with supports on the same level can be solved without simultaneous equations since the horizontal reactions pass through the same point.

If the system is asymmetric then summing moments about one reaction will give the vertical component of the other reaction.

To find the horizontal reaction component, cut a FBD at a point of known location (both x and y). So summing moments at C then finds the horizontal force.



$$\Sigma M_{@R2} = R1_V(40') - 6^K(36') - 1^K(24') - 3^K(4') = 0$$
$$R1_V(40') = 216^{K\text{-}'} + 24^{K\text{-}'} + 12^{K\text{-}'} = 252^{K\text{-}'}$$
$$R1_V = 6.3^K \uparrow$$

$$\Sigma M_{@P2} = 6.3^K(16') - R1_H(20') - 6^K(12') = 0$$
$$R1_H(20') = 100.8 - 72 = 28.8^{K\text{-}'}$$
$$R1_H = 1.44^K \leftarrow$$

$$\Sigma F_H = 0 = -1.44^K + R2_H$$
$$R2_H = 1.44^K \rightarrow$$

$$\Sigma F_V = 0 = 6.3 - 6 - 1 - 3 + R2_V$$
$$R2_V = 3.7^K \uparrow$$

$$F_a = \sqrt{6.3^2 + 1.44^2} = 6.46^K \ T$$

$$F_d = \sqrt{3.7^2 + 1.44^2} = 3.97^K \ T$$

FIND A

$$\frac{R1_V}{R1_H} = \frac{6.3}{1.44} : \frac{A}{4}$$
$$A = 17.5'$$

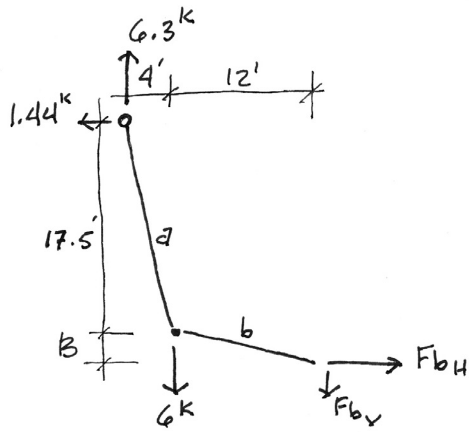
FIND D

$$\frac{3.7}{1.44} : \frac{D}{4}$$
$$D = 10.27'$$

Cables Forces

Ex 2 - even supports – asymmetric loading

FBD



$F_a = \sqrt{6.3^2 + 1.44^2} = 6.46^k$ T

$F_d = \sqrt{3.7^2 + 1.44^2} = 3.97^k$ T

FIND A

$\frac{R1_V}{R1_H} = \frac{6.3}{1.44} : \frac{A}{4}$

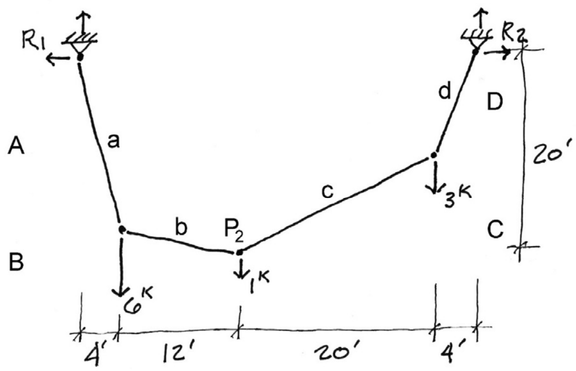
$A = 17.5'$

FIND D

$\frac{3.7}{1.44} : \frac{D}{4}$

$D = 10.27'$

Full diagram



$\Sigma M_{@R2} = R1_V(40') - 6^k(36') - 1^k(24') - 3^k(4') = 0$

$R1_V(40') = 216^{k\text{-}'} + 24^{k\text{-}'} + 12^{k\text{-}'} = 252^{k\text{-}'}$

$R1_V = 6.3^k \uparrow$

FBD $\Sigma M_{@P2} = 6.3^k(16') - R1_H(20') - 6^k(12') = 0$

$R1_H(20') = 100.8 - 72 = 28.8^{k\text{-}'}$

$R1_H = 1.44^k \leftarrow$

$\Sigma F_H = 0 = -1.44^k + R2_H$

$R2_H = 1.44^k \rightarrow$

$\Sigma F_V = 0 = 6.3 - 6 - 1 - 3 + R2_V$

$R2_V = 3.7^k \uparrow$

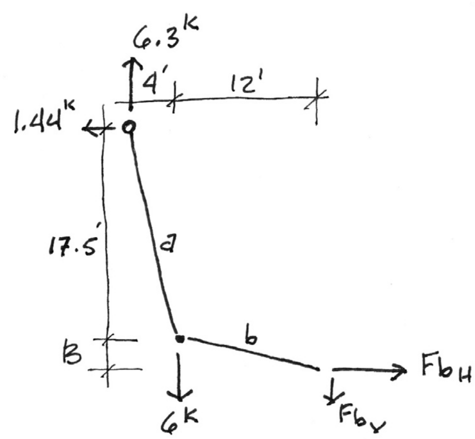
Cables Forces

Example 2

even supports – asymmetric loading

Proceed working inward from the reactions by cutting a FBD just before the end of the next section.

Because the force is in the same axis as the cable and there is no flexure in the cable, the ratio of the force components is proportional to the slope of the cable.



$\Sigma F_V = 0 = 6.3^k - 6^k - Fb_V$

$Fb_V = 0.3^k$

$\Sigma F_H = 0 = -1.44^k + Fb_H$

$Fb_H = 1.44^k$

$F_b = \sqrt{0.3^2 + 1.44^2} = 1.47^k$ T

FIND B

$\frac{0.3}{1.44} : \frac{B}{12}$ $B = 2.5'$

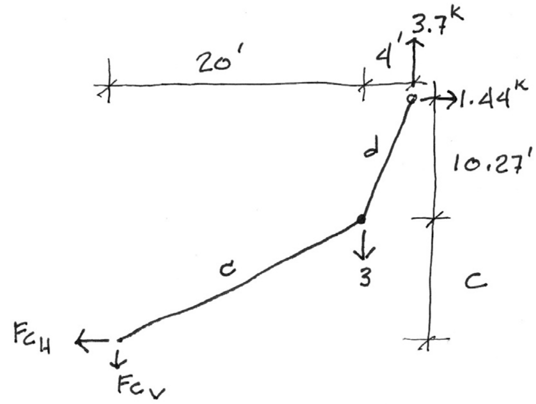
Cables Forces

Example 2

even supports – asymmetric loading

Proceed working inward from the reactions by cutting a FBD just before the end of the next section.

Because the force is in the same axis as the cable and there is no flexure in the cable, the ratio of the force components is proportional to the slope of the cable.



FIND C

$$\frac{0.7}{1.44} : \frac{C}{20'} \qquad C = 9.72'$$

$$\Sigma F_V = 0 = 3.7^K - 3^K - F_{cV}$$
$$F_{cV} = 0.7^K$$

$$\Sigma F_H = 0 = 1.44^K - F_{cH}$$
$$F_{cH} = 1.44^K \rightarrow$$

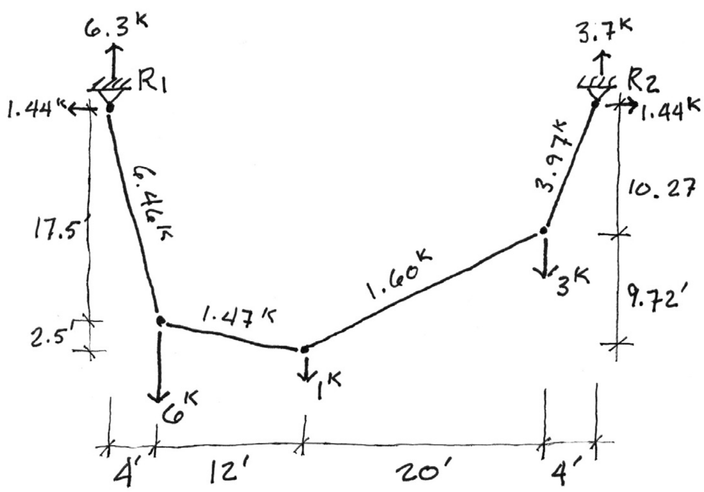
$$F_c = \sqrt{0.7^2 + 1.44^2} = 1.60^K \ T$$

Cables Forces

Example 2

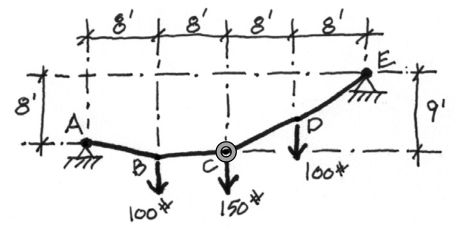
even supports – asymmetric loading

The final diagram compares the tension in each of the cable segments. Because of the asymmetry of the loading, the forces are also asymmetric. Note also that they generally increase toward the reactions.



Cables Forces

Example 3
uneven supports



The analysis of a cable system is made by cutting a series of FBDs joint by joint. Each joint is analyzed as a concentric force system. All forces must sum to zero.

$$\sum Fx = 0 \quad \sum Fy = 0 \quad \sum M = 0$$

To define the shape, the end points and one other point are needed.

The procedure usually begins with the summation of moments to determine the end reactions.

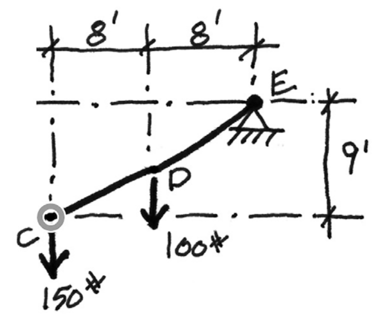
$$\Sigma M@A = 0$$
$$100^{\#}(8') + 150^{\#}(16') + 100^{\#}(24') - E_Y(32') + E_x(8') = 0$$
$$E_Y(32') = E_x(8') + 5600^{\#\text{-}'}$$
$$E_Y = E_x(0.25') + 175^{\#\text{-}'}$$

Since the location of point C is known, a FBD is cut at C of the right side. Summing moments at C will give a second equation for Ex and Ey.

$$\Sigma M@C = 0$$
$$100^{\#}(8') - E_Y(16') + E_x(9') = 0$$
$$E_Y(16') = E_x(9') + 800^{\#\text{-}'}$$
$$E_Y = E_x(0.5625') + 50^{\#\text{-}'}$$

Cables Forces - uneven supports

Example 3



Setting the two equations equal to Ey, the value is found for Ex.

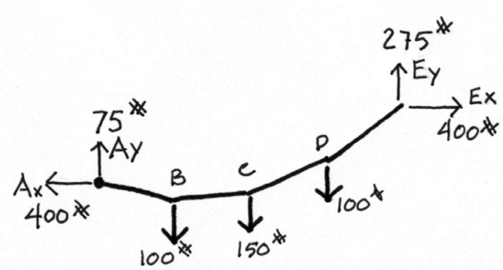
Then using Ex, Ey can be solved.

Finally Ax and Ay are found by summing forces.

$$E_x(0.25) + 175 = E_x(0.5625) + 50$$
$$125 = E_x(0.3125)$$
$$E_x = 400^{\#} \rightarrow$$

$$E_Y = E_x(0.5625) + 50$$
$$E_Y = 400^{\#}(0.5625) + 50$$
$$E_Y = 275^{\#} \uparrow$$

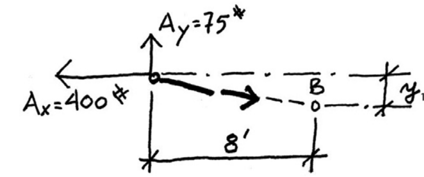
$$\Sigma F_Y = 0 = A_Y - 100^{\#} - 150^{\#} - 100^{\#} + 275^{\#}$$
$$A_Y = 75^{\#} \uparrow$$
$$\Sigma F_x = 0 = -A_x + 400$$
$$A_x = 400^{\#} \leftarrow$$



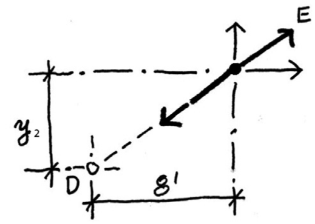
Cables Forces: Ex. 3 - uneven supports

Working from the reactions inward, the slope of each segment is determined by the components of the force. The remaining forces can be found using the FBD of the joint..

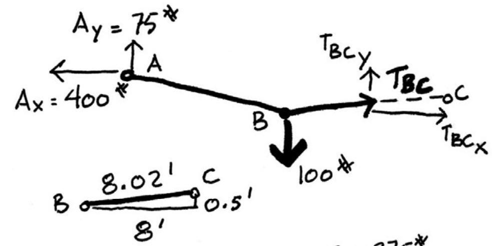
FBD-1



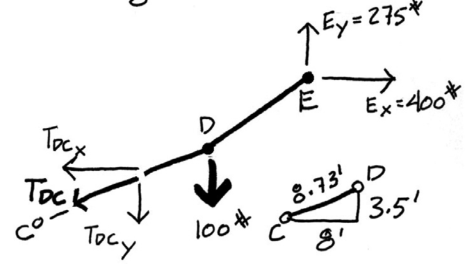
FBD-2



FBD-3



FBD-4



FBD1

$$\frac{400}{75} : \frac{8}{y_1} \qquad y_1 = 1.5'$$

FBD2

$$\frac{400}{275} : \frac{8}{y_2} \qquad y_2 = 5.5'$$

FBD3

points B and C are both known. $\text{Slope} = \frac{0.5}{8}$

$$\sum Fy = 0 = 75 - 100 + Tbcy \qquad Tbcy = 25\#$$

$$\sum Fx = 0 = \quad -400 + Tbcx \qquad Tbcx = 400\#$$

FBD4

points C and D are both known. $\text{Slope} = \frac{3.5}{8}$

$$\sum Fy = 0 = 275 - 100 - Tdey \qquad Tdey = 175\#$$

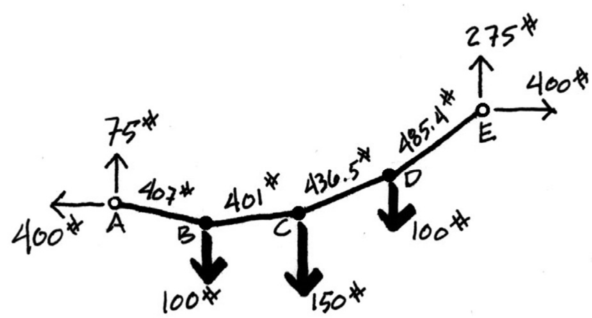
$$\sum Fx = 0 = 400 - Tdex \qquad Tdex = 400\#$$

Cables Forces – uneven supports
Example 3

Using FBDs of each joint in turn the force components and slopes can be determined.

Cable forces are found by adding the components with Pythagorean formula.

In summary the cable forces are:
Tab = 407.0 #
Tbc = 400.8 #
Tcd = 436.5 #
Tde = 485.4 #



Cable Structures

Munich Olympic Buildings
Frei Otto with Günter Behnisch, 1972

Institute for Lightweight Structures
Frei Otto, 1965

Cable Bracing

Grain silos in Michigan

Messe Tower in Leipzig
Engineer: Schlaich Bergernam and Partners
Architect: Gerkan, Marg and Partners
Photo by Prolineserver

Cable Structures

National Stadium Brasilia
photo by Tomás Faquini