Combined Stress

- Tension + Flexure
- Compression + Flexure
- Eccentric Loads

Axial Stress

- Loads pass through the centroid of the section, i.e. axially loaded
- Member is straight
- Load less than buckling load

Then:

\[ f_a = \frac{P}{A} \]
**Flexural Stress**

- Loads pass through the **centroid** of the section
- Member is **straight**
- Member deflects in the plane of loading (vertical) – no lateral tensional buckling (LTB)

\[ f_b = \frac{Mc}{I} \]

---

**Axial + Flexure**

Axial Tension + Flexure

Stress addition by sign:
- tension + tension = total
- tension − compression = total

\[ f = \frac{P}{A} \pm \frac{Mc}{I} \]
Axial + Flexure

Axial Compression + Flexure

The deflection caused by flexure together with the axial compression results in a secondary moment

\[ M_2 = P \Delta \]

\[
f = \frac{P}{A} \pm \frac{Mc_x}{I_x} \pm \frac{P\Delta c_x}{I_x}
\]

Second Order Stress

“P Delta Effect”

1. Eccentric load causes bending
2. Deflection increases eccentricity
3. Eccentric load results in Pe moment
4. Moment causes additional bending
**Eccentric Loads**

- Load offset from centroid
- \( M_e = P \cdot e \)
- Total load = \( P + M_e \)

combined stress (interaction) formula:

\[
f = \frac{P}{A} \pm \frac{M_e}{I} \cdot c
\]

\[
\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0
\]

**Combined Stress**

- Stresses combine by superposition
- Values add or subtract by sign
Bi-axial Flexure

\[ f = \frac{P_1}{A} \pm \frac{M_x}{S_x} + \frac{P_2}{A} \pm \frac{M_y}{S_y} \]

Example of combined Stress
Beam Columns
Other Examples

Combined Shear and Bending Stress
Principal Stresses
Principal Stresses

The surfaces of maximum tension and maximum compression stresses are at right angles, 90°.

Given the normal and shear stresses on the faces of any elemental square, the principal normal stresses can be calculated by:

\[
S_{N_{\text{max}}} = \frac{S_x + S_y}{2} + \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + S_s^2}
\]

\[
S_{N_{\text{min}}} = \frac{S_x + S_y}{2} - \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + S_s^2}
\]

\[
\tan 2\theta = -\frac{2S_s}{S_x - S_y}
\]

Example (by equations)

\[
S_{N_{\text{max}}} = \frac{S_x + S_y}{2} + \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + S_s^2}
\]

\[
S_{N_{\text{MAX}}} = \frac{16 - 10}{2} + \sqrt{\left(\frac{6 + 10}{2}\right)^2 + 0^2} = 8
\]

\[
S_{N_{\text{min}}} = \frac{S_x + S_y}{2} - \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + S_s^2}
\]

\[
S_{N_{\text{MIN}}} = \frac{16 - 10}{2} - \sqrt{\left(\frac{6 + 10}{2}\right)^2 + 0^2} = -12
\]
Mohr’s Circle – Graphic Method to find Principal Stress
1. Choose two adjacent sides of the elemental square (H & V)
2. Plot the coordinates \((s_y, s_y)\) and \((s_x, s_x)\) with \(S_N\) as abscissa and \(S_s\) as ordinate.
   Take normal tension stress and clockwise shear stress as positive.
3. Connect the two points with a line and find the center, C
4. Draw a circle with center at C, passing through H and V
5. Calculate \(\tan 2\theta = FV/CF\)
6. Read principal stress values at A and B and max shear stress at D

Mohr’s Circle – Example

\[ OB = -OC + \text{radius of circle} = -OC + \sqrt{(CF)^2 + (FV)^2} \]
\[ = -2 + \sqrt{(8)^2 + (6)^2} = -2 + 10 \]
\[ = 8; \]
\[ OA = -OC - \text{radius of circle} = -OC - \sqrt{(CE)^2 + (EH)^2} \]
\[ = -2 - 10 \]
\[ = -12 \text{ ksi;} \]
\[ \tan 2\theta = \frac{FV}{CF} = \frac{8}{6} = 0.75, \quad 2\theta = 36.8^\circ, \quad \theta = 18.4^\circ \]
\[ s_{\text{max}} = 10 \text{ ksi} \]
\[ 2\theta_s = 2\theta + 90^\circ = 36.8^\circ + 90^\circ = 126.8^\circ \]
\[ \theta_s = 63.4^\circ \]
Principal Stresses

Pier Luigi Nervi, Gatti Wool Factory, Rome

Lines of principle stress

Pier Luigi Nervi, Palace of Labor Floor System
Palace of Labor (Palazzo del Lavoro)

The Ribbed Floor Slab Systems of Pier Luigi Nervi; Allison B. Halpern, David P. Billington, Sigrid Adriaenssens in “Beyond the Limits of Man” IASS Symposium 2013